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A COMPUTATIONAL PROGRAM AND
EXTENDED TABULATION OF LEGENDRE FUNCTIONS
OF SECOND KIND AND HALF ORDER

by

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ABSTRACT

This report presents a tabulation of Legendre functions of second kind and half integral order, $Q_{n-\frac{1}{2}}(z)$, for orders from ½ through 10½, with an accuracy of one unit variation in the fifth digit. The arguments are given by $z = (1+y^2)$ with $\Delta y = 0.01$ and their range extends from 1.0001 to the terminal argument at which the function is equal to or less than 0.00001. Also included is a tabulation of $Q_{n-\frac{1}{2}}(z)$ for the same range of order and argument where the argument is given by $z = (1+y)$ with $\Delta y = 0.1$. A general description of the computational program and program flow charts is included.

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SYMBOLS

a_{np}, b_{np}	coefficients in the expansion for $Q_{n-\frac{1}{2}}$
C	complex contour of integration
$G_n(k^2)$	Riegels function
i	$(-1)^{\frac{1}{2}}$
n, p	real integer numbers
$P_n(z)$	Legendre function of first kind
$Q_n(z)$	Legendre function of second kind
t	dummy variable of integration
u	a solution of the Legendre equation
z	argument of Legendre function
δ_{n0}	Kronecker delta
$\Gamma(\ )$	Gamma function

A COMPUTATIONAL PROGRAM AND
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OF SECOND KIND AND HALF ORDER

INTRODUCTION

The preparation of these extended tables of the Legendre function of second kind and half order is an outgrowth of our theoretical study of three-dimensional flow through ducted propellers. Its origin is connected with the determination of the velocity field induced by the complete configuration[1], see Fig. 0.1. In our particular model, radial vortex lines, circular vortex rings and semi-infinite helical vortices replace the propeller blades, the duct and their shed vorticity, respectively. The velocity fields generated by these vortices are found from integrations of the Biot-Savart law.

In other related studies similar integrations of the Biot-Savart law have been performed with results expressed in terms of elliptic functions and/or Bessel functions. For example, D. Küchemann and J. Weber[2] have determined the velocity field of a constant-strength circular vortex ring in terms of complete elliptic integrals. T. Moriya[3] has integrated the Biot-Savart law along a helical path, finding the velocity field due to shed vortices from the propeller as integrals of Bessel functions.

In the present study, a great simplification was possible by the Fourier analysis of the velocity fields of both propeller and shroud[1]. This yielded expressions in terms of the

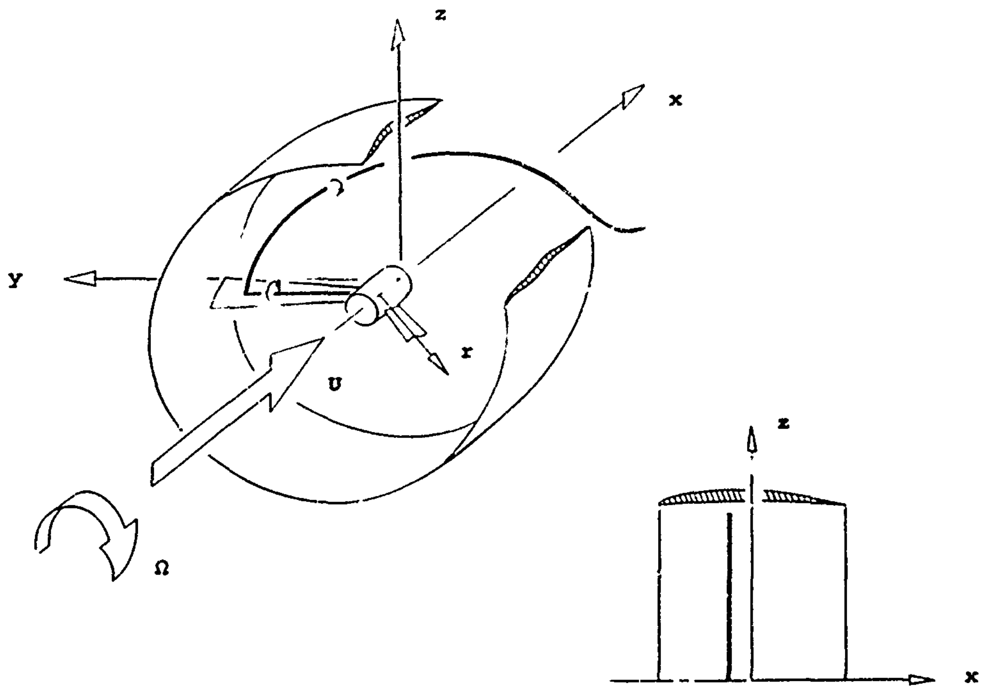


FIGURE 0.1

GENERAL DUCTED PROPELLER CONFIGURATION[1]

Legendre functions of second kind and half integral order. Of foremost importance, however, was the bringing into focus of the pertinent physical properties of these components. The order $(n-\frac{1}{2})$ of the Legendre function appears as $(mN-\frac{1}{2})$ where m is the number determining the rank of the term in the Fourier series expansion and N the number of propeller blades. The arguments, in general, become functions of the distances between the field point and the vortex elements and lie in the interval from plus one to positive infinity. For all orders in this range, the functions have a logarithmic singularity at an argument of plus one and decay monotonically to zero at positive infinity. For a given argument, the further monotonic decay with increasing order from minus one half facilitates the study of the higher harmonics of the velocity field.

In order to carry out, on a Burroughs 220 Complex, the calculations required by the analysis, provisions for the computation and storing of the Legendre functions were required. A sub-project was initiated, aimed at establishing a computational routine for the half-integer Legendre functions of the second kind and their subsequent tabulation as part of the overall program. The criteria selected were that a prescribed accuracy of not more than one digit error in the fifth decimal place be maintained throughout the range of calculation and that the sequence of arguments be such that three point interpolation yield the same accuracy. Consistent with

these criteria, values of the Legendre functions less than 0.00001 were not calculated.

A general discussion of the properties and behavior of the Legendre functions is contained in Chapter One. The important features of the computational program and the checking procedure follow in Chapter Two. Finally, the detail computational flow charts and tables are presented in Chapters Three and Four, respectively.

The National Bureau of Standards Tables of Associated Legendre Functions[4] provide values of the Legendre functions of the second kind and half order for arguments from one to ten in increments of 0.1, and for orders from minus one-half to four and one-half. It is believed that the interim tables published herein fill, at present, an important gap in the calculated values of these Legendre functions.

CHAPTER ONE

MATHEMATICAL RELATIONSHIPS

1.1 General Properties of the Legendre Function

It is known that the Legendre equation,

$$(1-z^2)\frac{d^2u}{dz^2} - 2z\frac{du}{dz} + n(n+1)u = 0 \tag{1.1}$$

is satisfied, in general, by an expression of the type

$$u = \frac{1}{2\pi i}\oint_C \frac{(1-t^2)^n}{2^n(z-t)^{n+1}}\,dt \tag{1.2}$$

provided that the contour of integration taken is such that

$$\frac{(1-t^2)^{n+1}}{(z-t)^{n+2}}$$

returns to its original value after completing the contour[5]. In the form above, Eq. (1.1) is a particular case of the Gegenbauer equation

$$(1-z^2)\frac{d^2u}{dz^2} - 2(1+\beta)z\frac{du}{dz} + n(n+2\beta+1)u = 0 \tag{1.3}$$

with $\beta = 0$.

One form of Eq. (1.2) is the Legendre function of the first kind, usually denoted by $P_n(z)$, namely

$$P_n(z) = \frac{1}{2\pi i\ 2^n} \oint_C \frac{(1-t^2)^n}{(z-t)^{n+1}}\, dt \tag{1.4}$$

where n is the unrestricted order and C is the contour of integration as shown in Ref. 6. For non-integer orders a cut is needed along the negative real axis from -1 to $-\infty$ in order to make $P_n(z)$ single valued.

A second form, with which this report is concerned, is the Legendre function of the second kind valid for half and integer n,

$$Q_n(z) = \frac{1}{2^{n+1}} \int_{-1}^{1} \frac{(1-t^2)^n}{(z-t)^{n+1}}\, dt \tag{1.5}$$

which is obtained from the general solution through a suitable path of integration[6]. If n is any positive integer or zero, Eq. (1.5) can be integrated in closed form, yielding

$$Q_0(z) = \tfrac{1}{2} \ln \left(\frac{z+1}{z-1}\right)$$

$$Q_1(z) - \tfrac{1}{2} z \ln \left(\frac{z+1}{z-1}\right) - 1 \text{ ; etc.} \tag{1.6}$$

In order to express $Q_n(z)$ in half integer orders, replace n by $n-\frac{1}{2}$ in Eq. (1.5), or

$$Q_{n-\frac{1}{2}}(z) = \frac{1}{2^{n+\frac{1}{2}}} \int_{-1}^{1} \frac{(1-t^2)^{n-\frac{1}{2}}}{(z-t)^{n+\frac{1}{2}}}\, dt \tag{1.7}$$

The singularity at $z = \pm 1$ for this form remains the same

as that for Eq. (1.6). An alternate expression for Eq. (1.7) can be written as[7]

$$Q_{n-\frac{1}{2}}(z) = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\cos 2n\tau}{(a^2 + 4\sin^2\tau)^{\frac{1}{2}}}\, d\tau \qquad (1.8)$$

where $z = 1 + \frac{a^2}{2}$. It is valid for all real arguments as well as for complex values of a except along the cut from $-2i$ to $-i\infty$ in the complex plane[7]. Eqs. (1.7) and (1.8) were found to be suitable for different ranges of the numerical calculations.

1.2 Series Representation for Large Values of the Argument

For large values of its argument, $Q_{n-\frac{1}{2}}(z)$ approaches zero asymptotically as z tends to infinity. To obtain an appropriate series the integrand in Eq. (1.5) is expanded in a power series in inverse powers of z which is uniformly convergent with respect to t , and the following hypergeometric series F is obtained.

$$Q_n(z) = \frac{(\pi)^{\frac{1}{2}}}{(2z)^{n+1}} \frac{\Gamma(n+1)}{\Gamma(n+3/2)} F\left(\frac{1}{2}+\frac{n}{2}\,,\ 1+\frac{n}{2}\,;\ n+\frac{3}{2}\,;\ 1/z^2\right) \qquad (1.9)$$

Replacing the hypergeometric series and setting n equal to $n-\frac{1}{2}$, one obtains:

$$Q_{n-\frac{1}{2}}(z) = 2^{n-\frac{1}{2}} \sum_{p=0}^{\infty} \frac{2^p\, \Gamma^2(n+p+\frac{1}{2})}{p!\ (2n+p)!} (z+1)^{-n-p-\frac{1}{2}} \qquad (1.10)$$

The behavior of this function for large argument is governed by the leading term thus, for $z \to \infty$

$$Q_{n-\frac{1}{2}}(z) \sim \frac{\pi}{2^{3n+\frac{1}{2}}} \frac{(2n)!}{(n!)^2} \frac{1}{z^{n+\frac{1}{2}}} \qquad (1.11)$$

1.3 Series Representation for Small Values of the Argument

As noted before the Legendre function of second kind has a logarithmic singularity at $z = \pm 1$. In the ducted propeller application we are interested only at the point $z = +1$. An appropriate series expression can be obtained by substituting a solution of the form

$$Q_{n-\frac{1}{2}}(z) = \sum_{p=0}^{\infty} a_{np}(z-1)^p + \ln(z-1) \sum_{p=0}^{\infty} b_{np}(z-1)^p \qquad (1.12)$$

into Eq. (1.1) with n set equal to $(n-\frac{1}{2})$. By comparison of the coefficients of powers of $(z-1)$ the following recurrence formulas for the coefficients a_{np} and b_{np} are:

$$b_{np+1} = b_{np} \frac{n^2-\frac{1}{4}-p(p+1)}{2(p+1)^2} \qquad (1.13)$$

$$a_{np+1} = a_{np} \frac{n^2-\frac{1}{4}-p(p+1)}{2(p+1)^2} - b_{np} \frac{2(n^2-\frac{1}{4})+(p+1)}{2(p+1)^3} \qquad (1.14)$$

The leading coefficients, a_{n0} and b_{n0}, can be determined[7] from the comparison of Eq. (1.12) with the expression for $Q_{n-\frac{1}{2}}(z)$ as given by Hobson[8]:

$$a_{n0} = (5/2)\,\ln\ 2 - 2[1 + 1/3 + 1/5 \ldots 1/2n-1]\ (1 - \delta_{n0})$$

$$b_{n0} = -\ 1/2 \tag{1.15}$$

The general behavior of these constants is shown on logarithmic scale in Figs. 1.1 and 1.2. Their values are tabulated in Tables 4.1 and 4.2 for $1 \leq n \leq 11$ and $0 \leq p \leq 9$.

1.4 Recurrence Formulas

Recurrence formulas, valid for Legendre functions of both the first and second kind are reproduced below for convenience[5]:

$$Q_{n-\frac{1}{2}}(z) = \frac{1}{2n}\left[\ Q'_{n+1/2}(z) - Q'_{n-3/2}(z)\ \right] \tag{1.16}$$

$$Q'_{n-\frac{1}{2}}(z) = \frac{n-\frac{1}{2}}{z^2-1}\left[\ zQ_{n-1/2}(z) - Q_{n-3/2}(z)\ \right] \tag{1.17}$$

$$Q_{n+\frac{1}{2}}(z) = \frac{2nz}{n+\frac{1}{2}}\,Q_{n-1/2}(z) - \frac{n-\frac{1}{2}}{n+\frac{1}{2}}\,Q_{n-3/2}(z) \tag{1.18}$$

$$Q'_{n+\frac{1}{2}}(z) = \frac{2nz}{n-\frac{1}{2}}\,Q'_{n-1/2}(z) - \frac{n+\frac{1}{2}}{n-\frac{1}{2}}\,Q'_{n-3/2}(z) \tag{1.19}$$

$$Q_{n-\frac{1}{2}}(z) = Q_{-n-\frac{1}{2}}(z) \tag{1.20}$$

$$Q'_{n-\frac{1}{2}}(z) = Q'_{-n-\frac{1}{2}}(z) \tag{1.21}$$

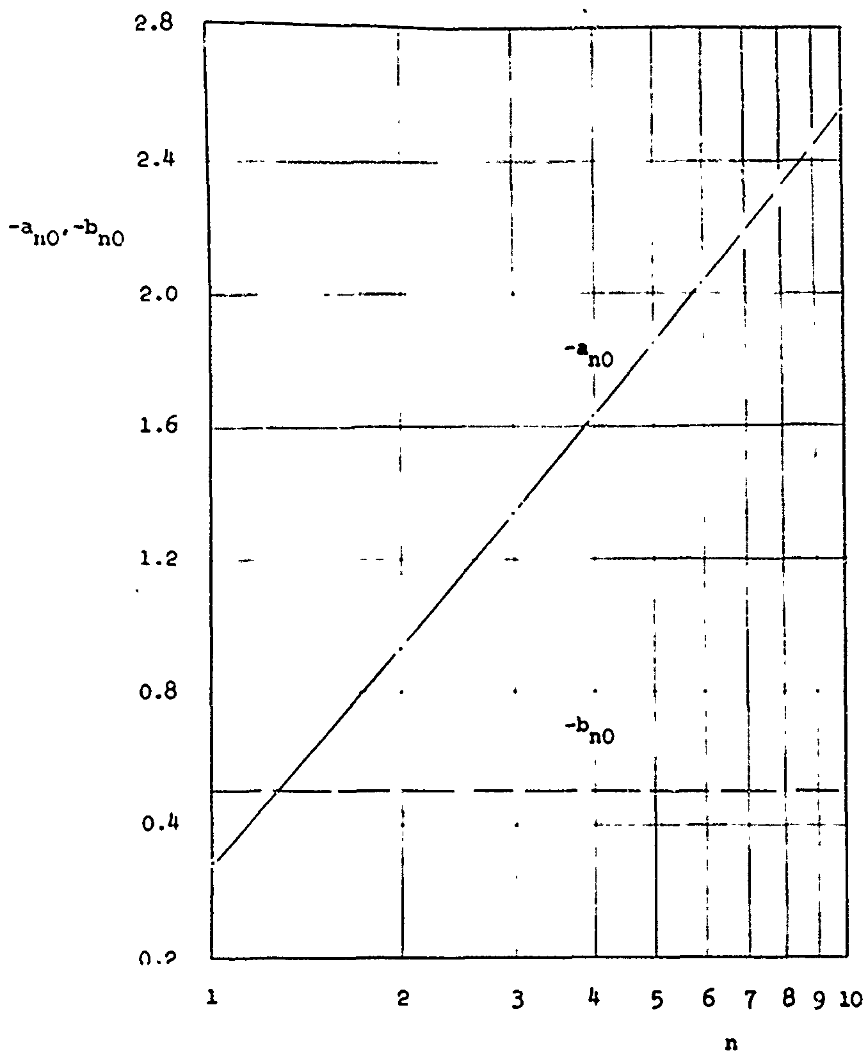


FIGURE 1.1

VARIATION OF THE COEFFICIENTS a_{n0} AND b_{n0} WITH ORDER n

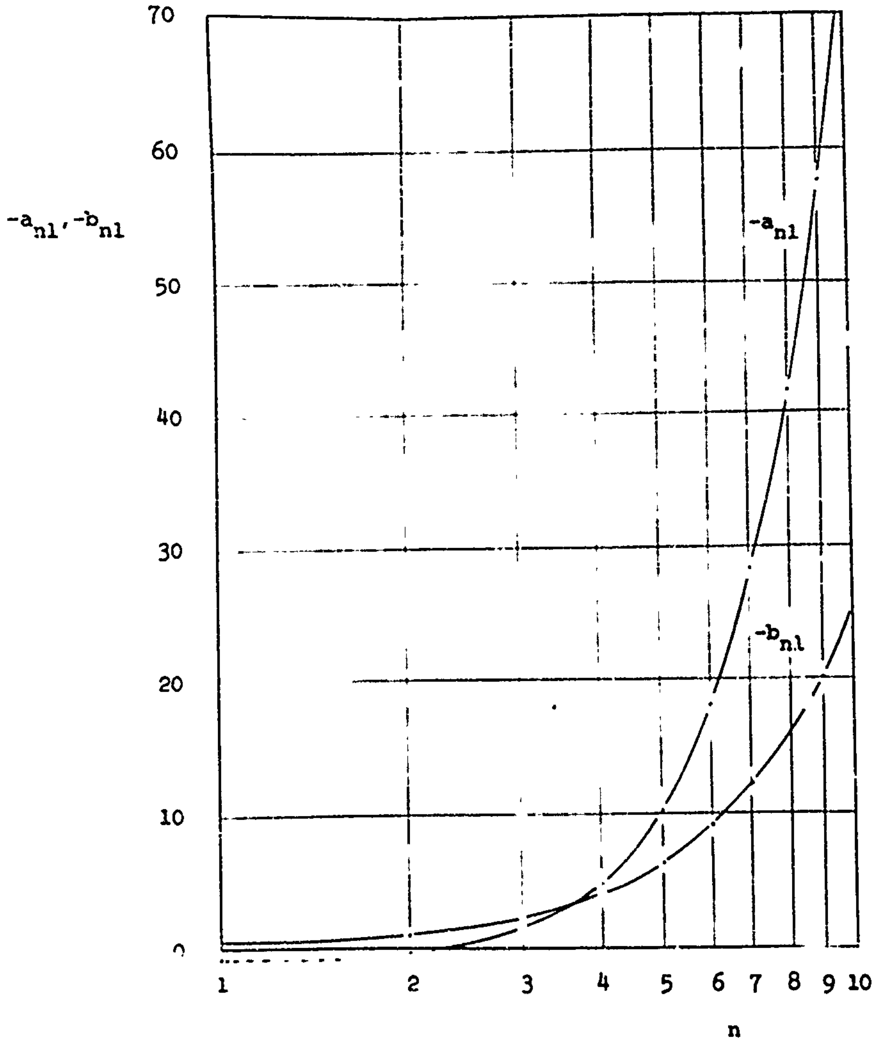


FIGURE 1.2

VARIATION OF THE COEFFICIENT a_{n1} AND b_{n1} WITH ORDER n

1.5 Relationship Between $Q_{n-\frac{1}{2}}(z)$ and Riegels Function

The derivative of $Q_{n-\frac{1}{2}}(z)$ is obtained from the differentiation of Eq. (1.8) with respect to the parameter a^2 , thus

$$Q'_{n-\frac{1}{2}}\left(1 + \frac{a^2}{2}\right) = -\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\cos 2n\tau}{(a^2+4 \sin^2 \tau)^{3/2}}\, d\tau \qquad (1.22)$$

where the prime denotes differentiation with respect to the argument. Eq. (1.22) can be related to the function $G_n(k^2)$ defined by F. Riegels[9,10] in his research on the aerodynamics of slender, quasi-axisymmetric bodies. This function is defined by

$$G_n(k^2) = (-1)^n \int_0^{\frac{\pi}{2}} \frac{\cos 2n\tau}{(1 - k^2 \sin^2 \tau)^{3/2}}\, d\tau \qquad (1.23)$$

and comparison with Eq. (1.22) yields:

$$Q'_{n-\frac{1}{2}}\left(1 + \frac{a^2}{2}\right) = -\tfrac{1}{4} \left[\frac{4}{a^2+4}\right]^{3/2} G_n \left[\frac{4}{a^2+4}\right] \qquad (1.24)$$

For small arguments; i.e., in the neighborhood of $k^2 = 1$, Riegels has expanded $G_n(k^2)$ in the form

$$k'^2\, G_n(k^2) = 1 + c_{n_2}\, k'^2 + d_{n_2}\, k'^2 \ln \frac{4}{k} \qquad (1.25)$$

$$+ \;.....$$

where $k'^2 = 1-k^2$. Although not given in analytical form, numerical values for the constants c_{n_2} and d_{n_2} were presented. Their analytical representation is possible, though, by series expansion of Eq. (1.24) and term by term comparison. Thus,

$$c_{n_2} = 4\,(n^2-\tfrac{1}{4})(1 + \tfrac{1}{3} + \ldots + \tfrac{1}{2n-1}) - (n^2+\tfrac{1}{4})$$

$$d_{n_2} = -2\,(n^2-\tfrac{1}{4}) \qquad (1.26)$$

A comparison of Riegels' coefficients as tabulated in Ref. 9 and as calculated from Eqs. (1.26) is shown in Table 4.3, for $0 \leq n \leq 7$.

CHAPTER TWO

COMPUTATIONAL PROCEDURES

2.1 General Discussion

For the numerical calculations required in the ducted propeller study a range of both order and argument was anticipated. The formulas presented in the previous chapter can be used to calculate numerically the values of the Legendre function of the second kind and half order over these ranges. In particular, for any order the integral representations are appropriate for moderate values of the argument and the series representations for the extreme values. Recurrence formulas in general were not considered desirable in view of error accumulation.

The argument for the main tabulation was generated by $z = 1+y^2$ where y varied in increments of 0.01. This provided satisfactory variation for low arguments where the value of the Legendre function is large, and yet converged fast enough toward higher arguments where the function is considerably smaller. In addition, another set of values was calculated by using $z = 1+y$ with $y = 0.1$. Values for both variations of argument are tabulated in Chapter Four.

2.2 Computation of Small Arguments, $1.0001 \leq z \leq 1.0900$

For small values of the argument, a trial run was made using the specialized expression Eq. (1.12) for $1.1 \leq z \leq 1.95$. The desired accuracy was achieved by adding terms in the series

until the difference between two consecutive values was less than 0.00001. It was noted, though, that beyond a certain point, which varies with order and argument, the magnitude of the Legendre function showed a tendency to increase as the order increased. This erroneous behavior was attributed to error accumulation in the higher coefficients since they are of a recursive nature, see Eqs. (1.13) and (1.14). This clearly indicated that the series was not suitable whenever a large number of terms had to be included. By comparison with other results, an argument of 1.09 was the point chosen to designate the upper limit of applicability for Eq. (1.12).

For more critical criteria of accuracy this upper limit would be even closer to the singular point. The error accumulation in the coefficients given by Eqs. (1.13) and (1.14) could have been reduced by using a double precision technique, thus increasing the range of application of Eq. (1.12). However, for expediency, an alternate formulation was selected for the next segment of the argument spectrum.

2.3 Computation of Moderate Arguments, $1.0900 \leq z \leq 5.0000$

For this argument range, Eqs. (1.7) and (1.8) were used. The numerical integration process was based on the generalized Simpson rule

$$\int_A^B f(x)dx = \frac{h}{3}(f_0+4f_1+2f_2+4f_3+ \ldots +4f_{m-3}$$

$$+2f_{m-2}+4f_{m-1}+f_m) - \frac{mh^5}{180} f^{iv}(\xi) \qquad (2.1)$$

where m is the number of intervals, h is the uniform increment in the variable of integration, and

$$f_0 = f(A)$$

$$f_k = f(A + kh)$$

$$f_m = f(B)$$

with ξ being any value of the variable of integration between the limits of integration. If f^{iv} is continuous throughout the interval of integration (A,B), the error diminishes like $1/m^4$ as $m \to \infty$.

In general, Eq. (1.7) was preferred. However, the fourth derivative of the integrand is singular at $t = \pm 1$ for $0 \leq n \leq 4$, making this form unacceptable. On the other hand, Eq. (.18) does not have this singular behavior and, since it is acceptable, it was used for this range.

The form of Eq. (1.8) was correlated with the argument $z = 1+y^2$ by replacing $\frac{a^2}{2}$ with y^2. Doing this one has

$$Q_{n-\frac{1}{2}}(1+y^2) = 2\int_0^{\frac{\pi}{2}} \frac{\cos 2n\tau \, d\tau}{(2y^2 + 4\sin^2\tau)^{\frac{1}{2}}} \tag{2.2}$$

from changing the limits of integration since the expression is an even function of τ. In this form, the arguments can be correlated regardless of formula used.

The number of intervals needed to satisfy the prescribed

criterion of accuracy was determined at numerous discrete points covering the entire anticipated range of argument and order. This was accomplished by calculating the maximum value of the fourth derivative within the interval of integration for each of the arguments considered and setting the remainder term equal to 0.00001. To simplify the computing procedure, the argument spectrum was further separated into several sub-ranges, and the maximum number of intervals within each sub-range was used throughout that sub-range as indicated in Fig. 2.1. For convenience, the argument range of Eqs. (1.10) and (1.12) is also shown.

As an example, Eq. (1.8) required 300 intervals for the range $1.0900 \leq z \leq 1.2025$ while for the same argument range, Eq. (1.7) required 400 intervals. With either equation, the argument range was split into small overlapping sub-ranges and the number of intervals varied. The values of the function were compared in the overlapped regions in order to check their accuracy.

2.4 Computation of Large Arguments, $5.0000 \leq z < \infty$

The series formula of Eq. (1.10) was used for the calculation of large arguments. The computing range started at $z = 5.0000$ and continued up to that value of z for which $Q_{n-\frac{1}{2}}(z) \leq 0.00001$. This terminal argument is a function of order, decreasing very rapidly as the order increases.

2.5 Accuracy Checks

For the integral forms, calculated values corresponding

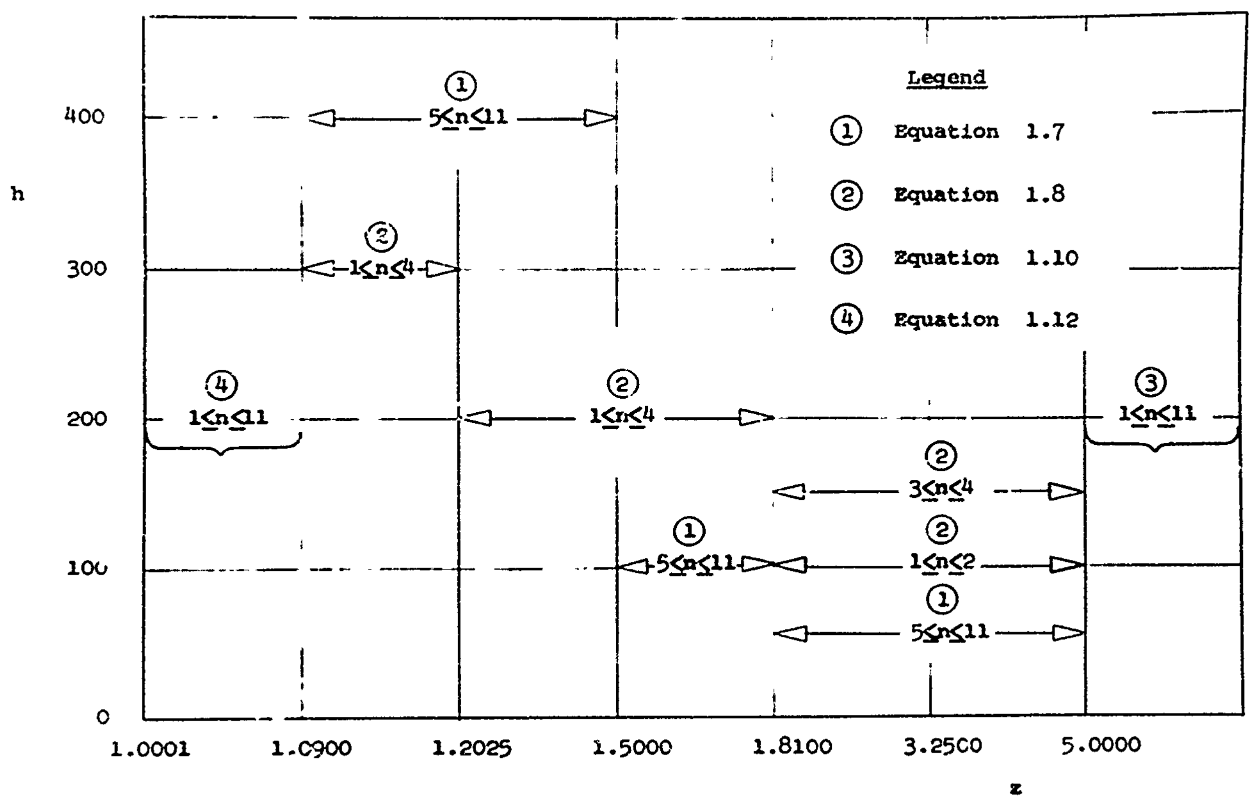


FIGURE 2.1

NUMBER OF INTERVALS h VS ARGUMENT RANGE z

to $1.1 \leq z \leq 5.0$ in increments of 0.1 for $0 \leq n \leq 5$ were compared with the available values from the N.B.S. tables[4] and agreement with each entry to the specified number of digits was found. In addition, three-point interpolation of the values at arguments given by $z = (1+y^2)$ were compared and checked with these values for $0 \leq n \leq 5$. For certain ranges of n above 5, the computation program was checked by varying the number of intervals. No changes are detected up to the fifth digit.

In the case of the series, terms were added, as previously noted, until two consecutive values of the resulting function did not differ by more than 0.00001.

2.6 Sub-routine Selection in the Master Program

The five sub-routines, corresponding to Eqs. (1.7), (1.8), (1.10), and (1.12) as well as an interpolation program for $1.0900 \leq z \leq 3.2500$ and $0 \leq n \leq 4$, were stored at appropriate locations. In the actual computation, the argument and order were calculated first. Proper branching was then introduced such that the program automatically selected the correct sub-routine.

CHAPTER THREE

FLOW CHARTS

In this chapter are included the flow charts representing the programming of the four equations used in the computation of the Legendre functions of second kind and half order.

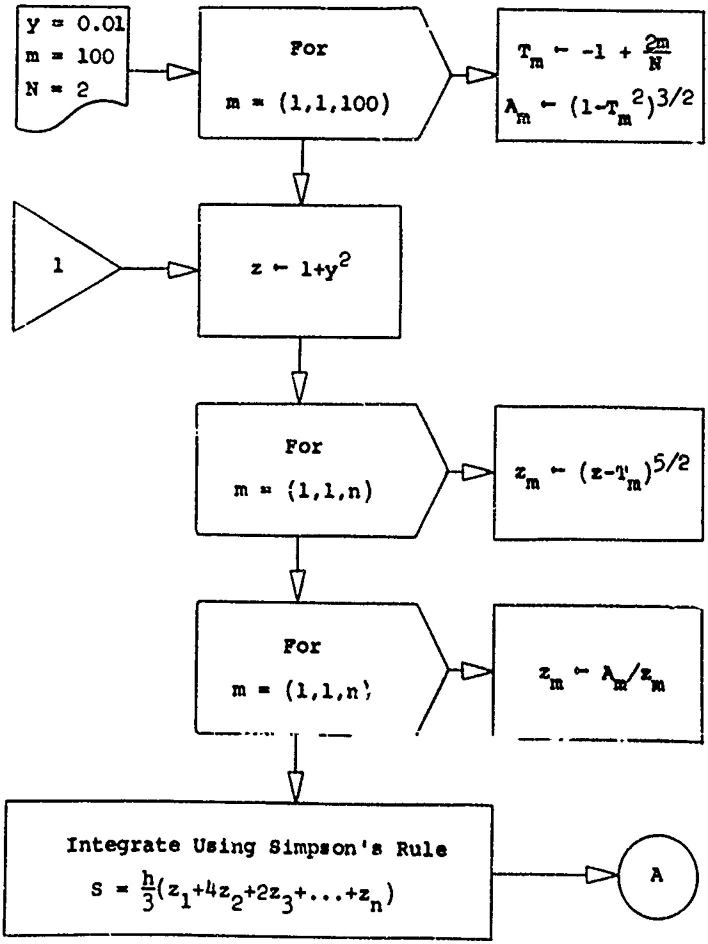
They are:

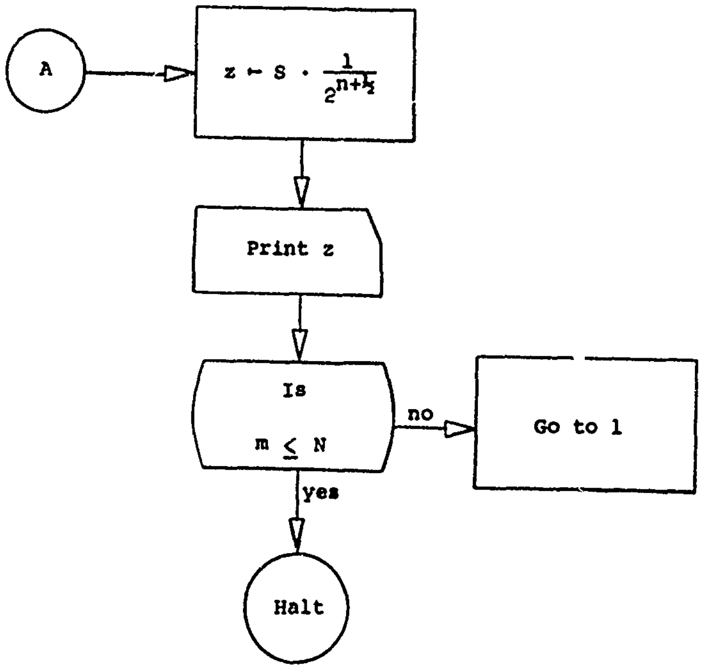
Eq. 1.7	Flow Chart No. 3.1
Eq. 1.8	Flow Chart No. 3.2
Eq. 1.10	Flow Chart No. 3.3
Eq. 1.12	Flow Chart No. 3.4

Quantities in the Flow Charts marked with an asterisk (*) are part of subsequent computations for which the Legendre functions are the input data.

PROGRAM FLOW CHART NO. 3.1

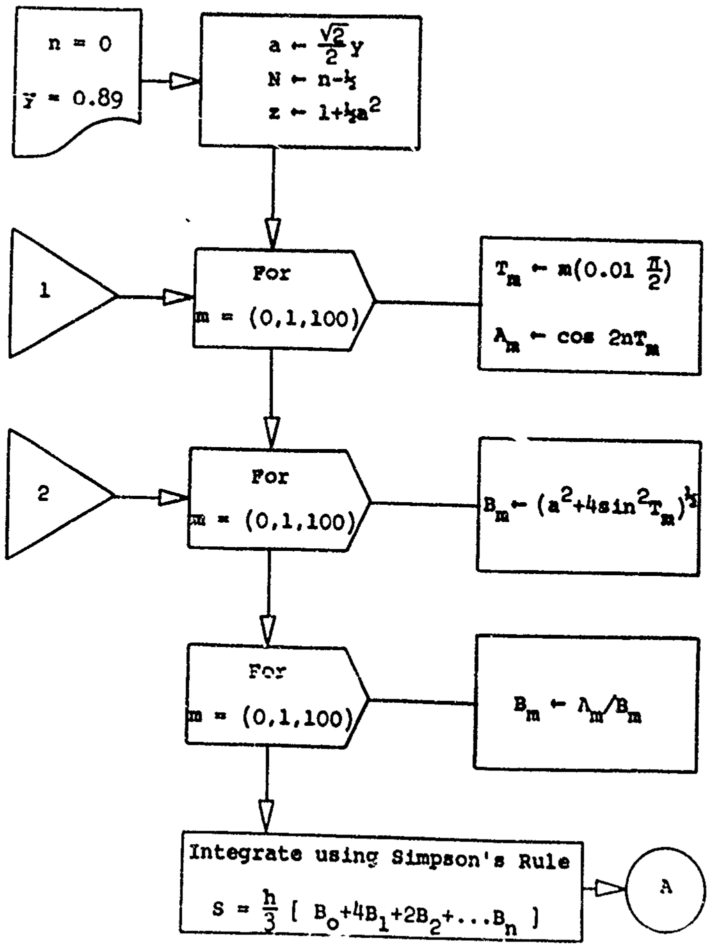
$$Q_{n-\frac{1}{2}}(z) = \frac{1}{2^{n+\frac{1}{2}}} \int_{-1}^{1} \frac{(1-t^2)^{n-\frac{1}{2}}}{(z-t)^{n+\frac{1}{2}}}\, dt$$

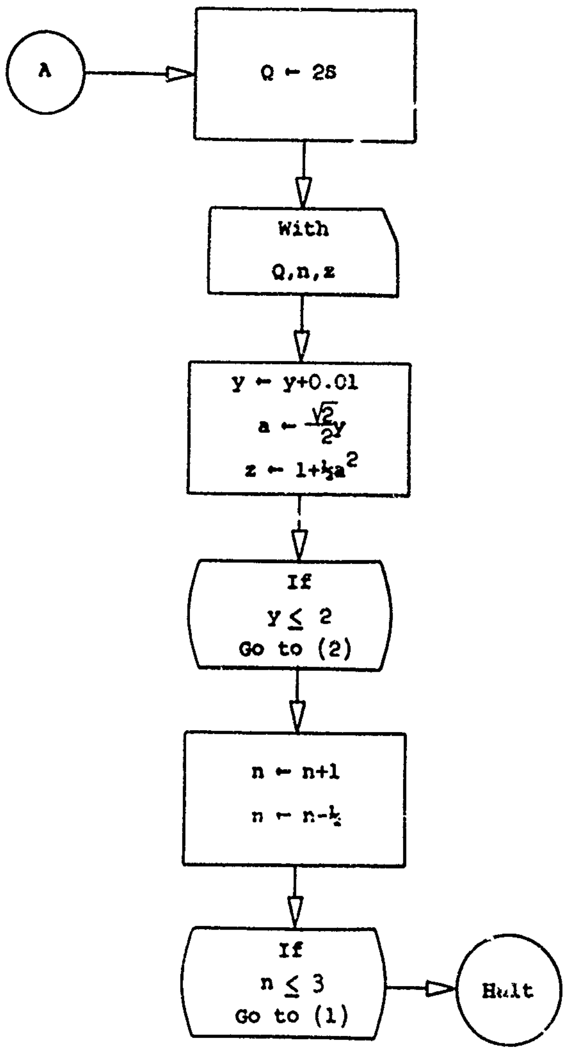



A
$z \leftarrow S \cdot \frac{1}{2^{n+\frac{1}{2}}}$
Print z
Is
$m \leq N$
no
Go to 1
yes
Halt

PROGRAM FLOW CHART NO. 3.2

$$Q_{n-\frac{1}{2}}\left(1+\frac{a^2}{2}\right) = 2\int_0^{\frac{\pi}{2}} \frac{\cos 2n\tau}{(a^2+4\sin^2\tau)^{\frac{1}{2}}}\,d\tau$$

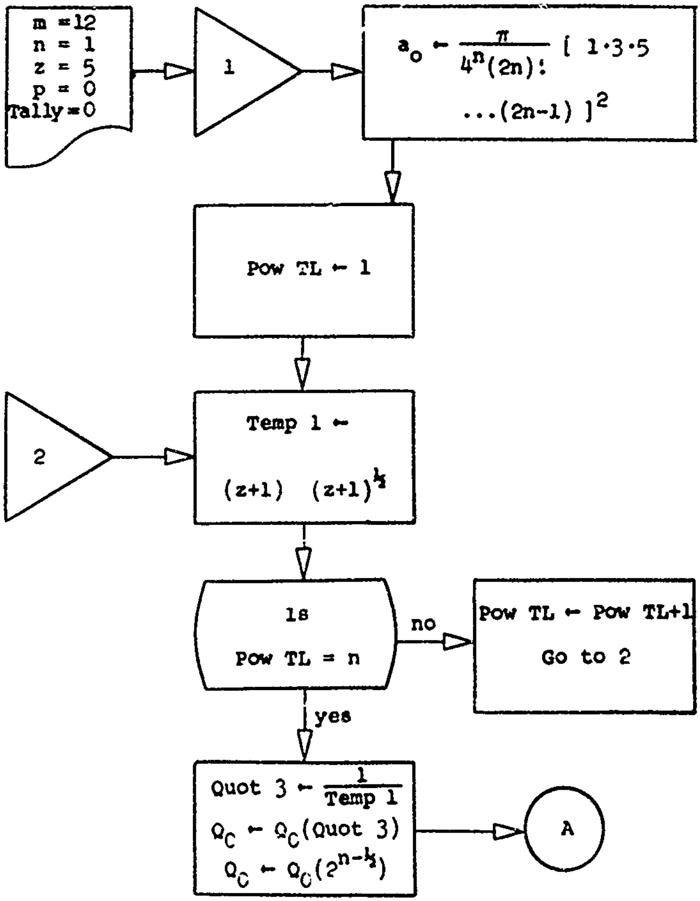


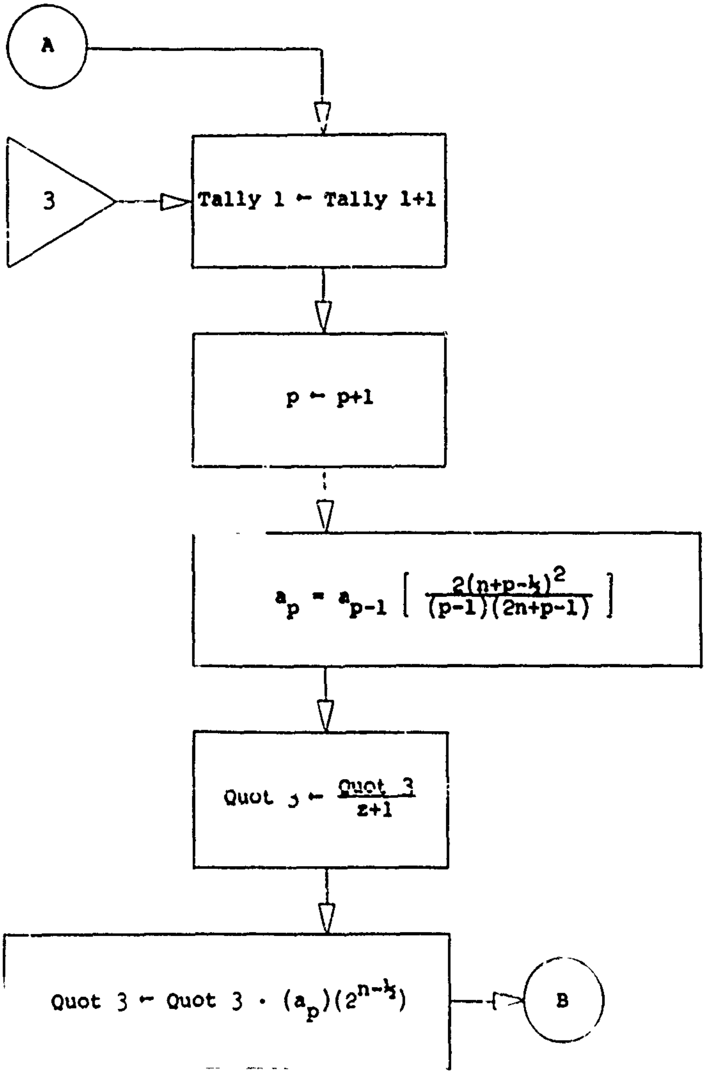


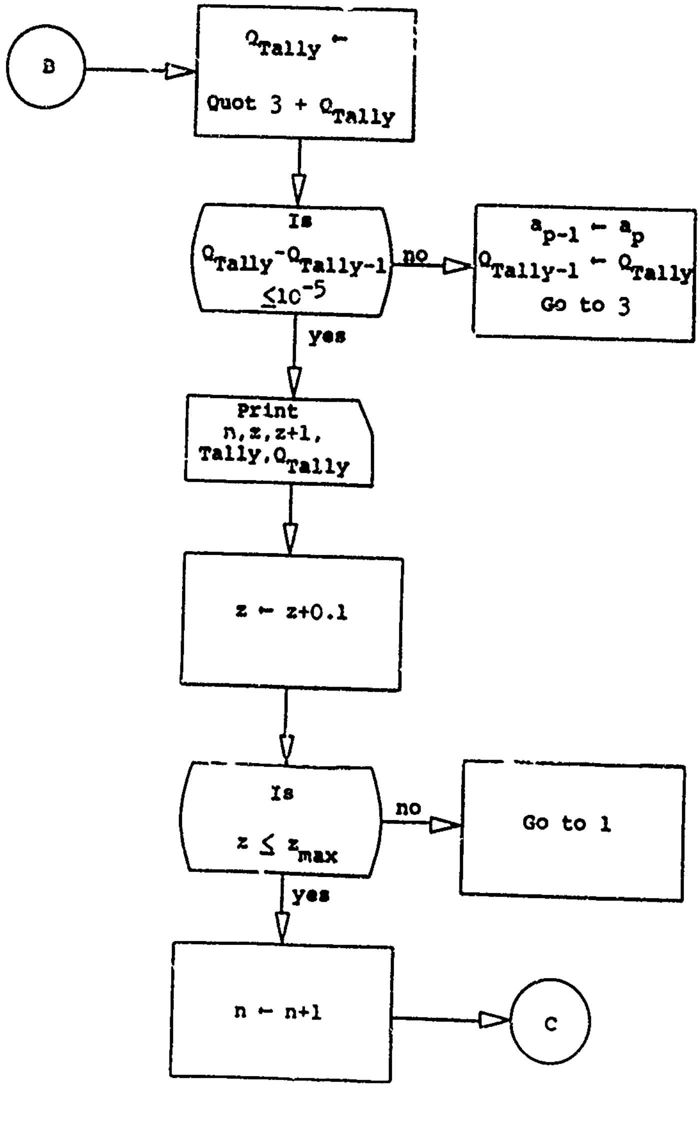
A
Q ← 2S
With
Q,n,z
y ← y+0.01
a ← $\frac{\sqrt{2}}{2}$y
z ← 1+½a^2
If
y ≤ 2
Go to (2)
n ← n+1
n ← n-½
If
n ≤ 3
Go to (1)
Halt

PROGRAM FLOW CHART NO. 3.3

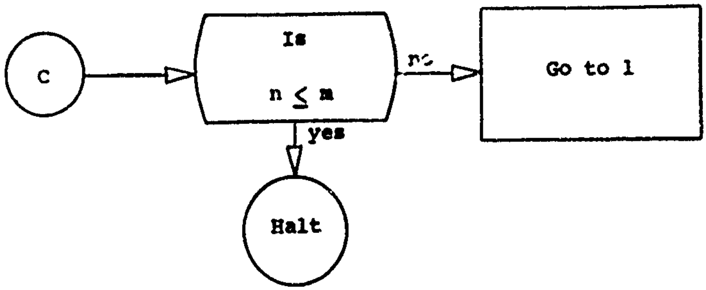
$$Q_{n-\frac{1}{2}}(z) = 2^{n-\frac{1}{2}} \sum_{p=0}^{\infty} \frac{2^p\, \Gamma^2(n+p+\frac{1}{2})}{p!\,(2n+p)!} (z+1)^{-n-p-\frac{1}{2}}$$




A
3
Tally 1 ← Tally 1+1
p ← p+1
$a_p = a_{p-1}\left[\frac{2(n+p-\frac{1}{2})^2}{(p-1)(2n+p-1)}\right]$
Quot 3 ← $\frac{\text{Quot 3}}{z+1}$
Quot 3 ← Quot 3 · $(a_p)(2^{n-\frac{1}{2}})$
B

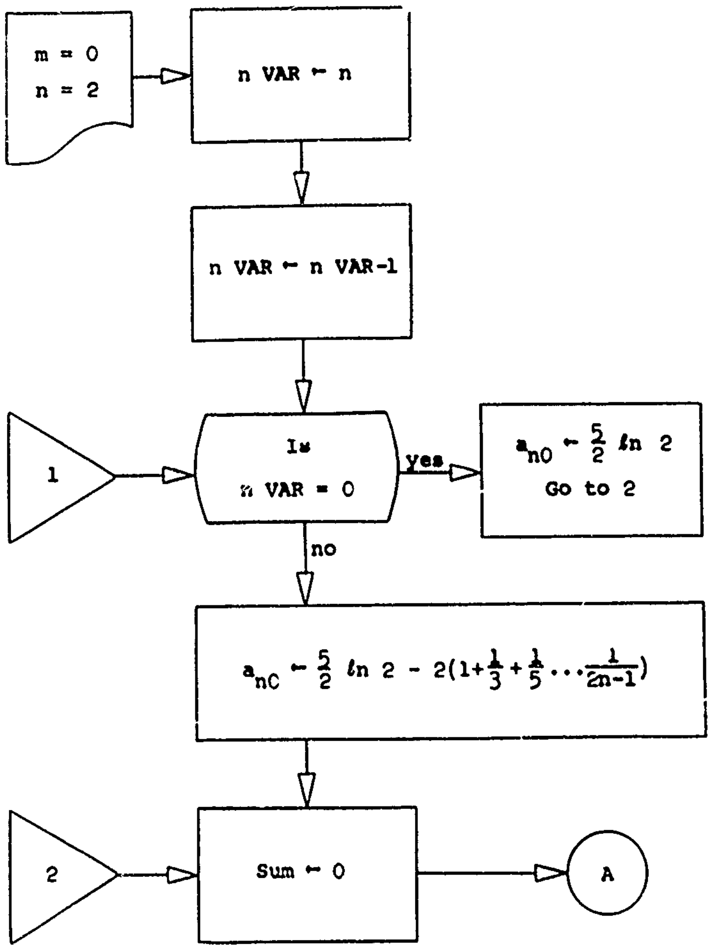


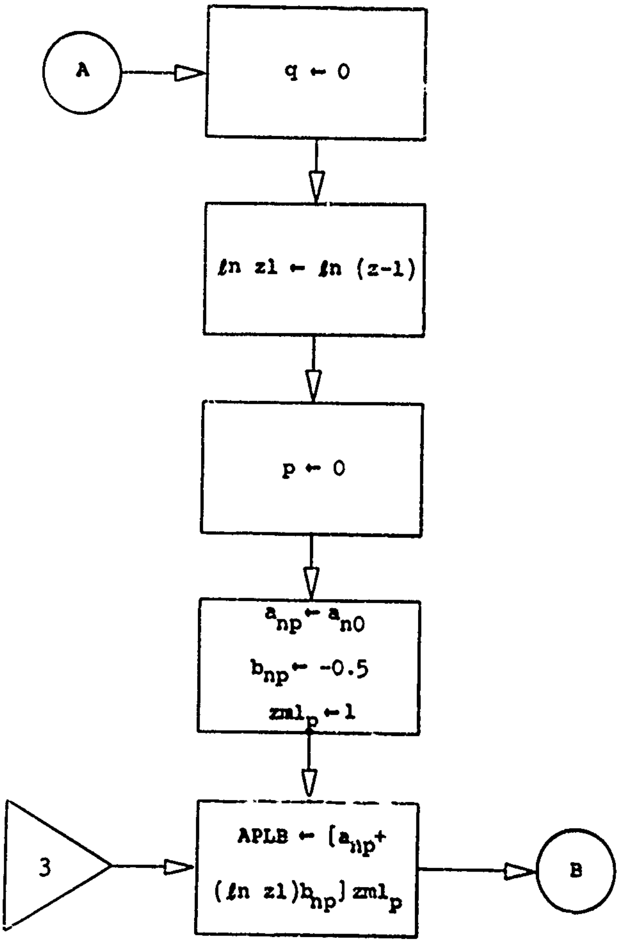
B
$Q_{Tally} \leftarrow$ Quot 3 + Q_{Tally}
Is $Q_{Tally} - Q_{Tally-1} \leq 10^{-5}$
no
$a_{p-1} \leftarrow a_p$
$Q_{Tally-1} \leftarrow Q_{Tally}$
Go to 3
yes
Print n, z, z+1, Tally, Q_{Tally}
$z \leftarrow z+0.1$
Is $z \leq z_{max}$
no
Go to 1
yes
$n \leftarrow n+1$
C

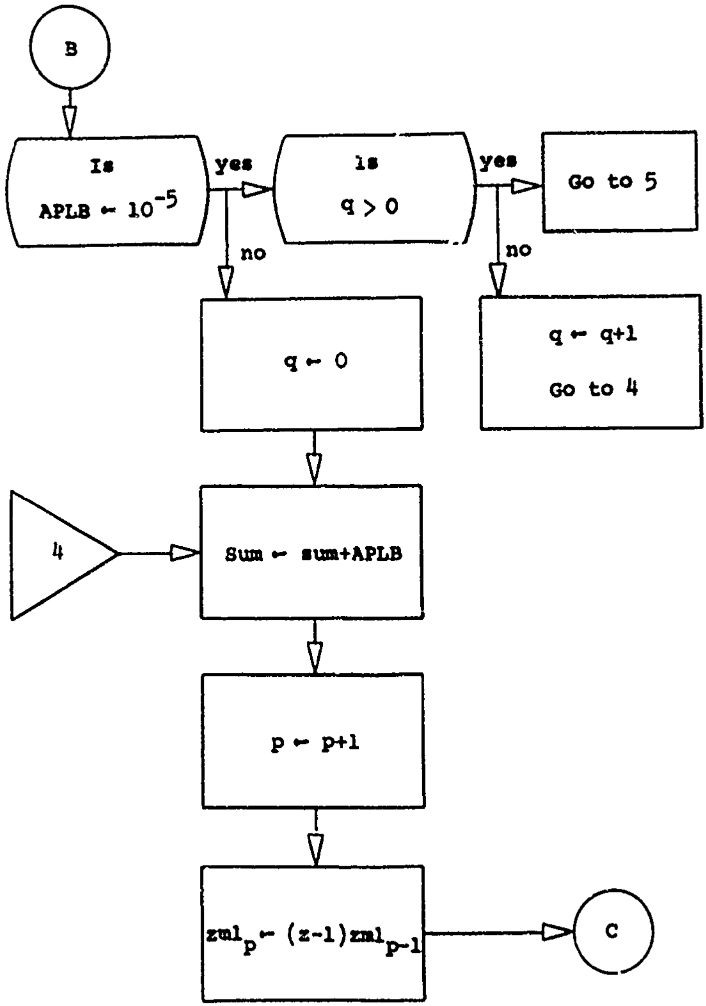

C
Is
n ≤ m
no
Go to 1
yes
Halt

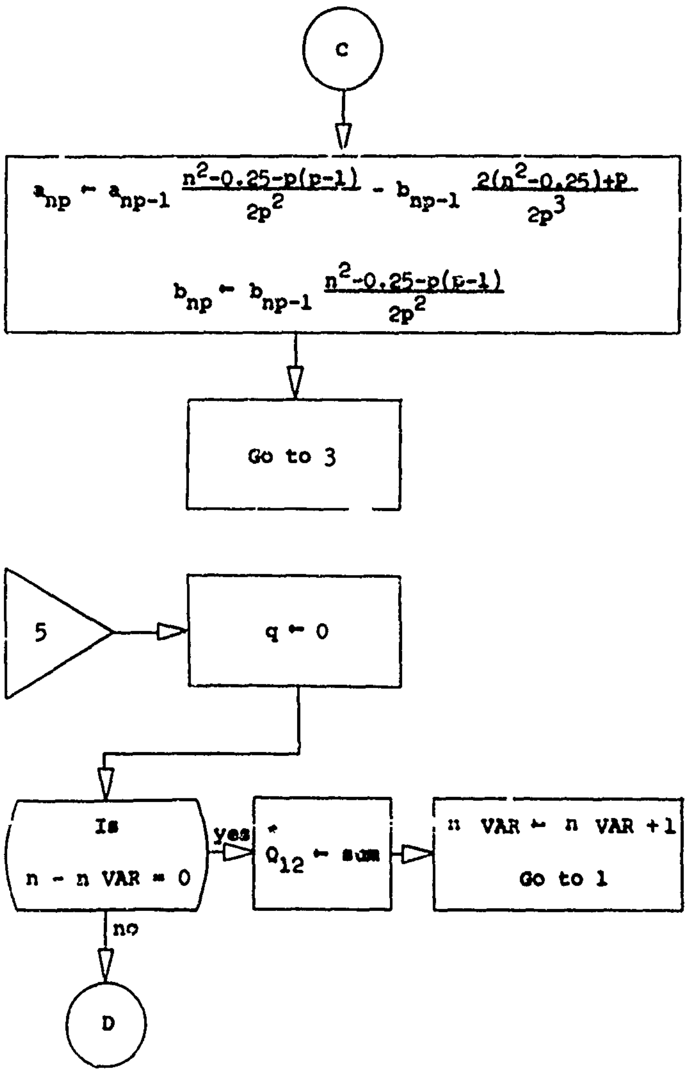
PROGRAM FLOW CHART 3.4

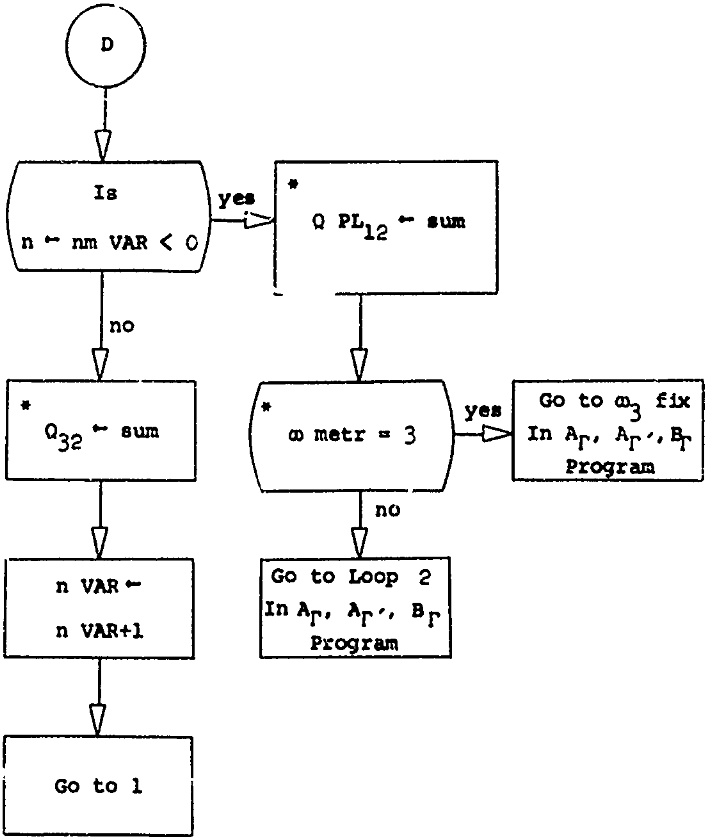
$$Q_{n-\frac{1}{2}}(z) = \sum_{p=0}^{\infty} a_{np}(z-1)^p + \ln(z-1)\sum_{p=0}^{\infty} b_{np}(z-1)^p$$




A
$q \leftarrow 0$
$\ell n\ z1 \leftarrow \ell n\ (z-1)$
$p \leftarrow 0$
$a_{np} \leftarrow a_{n0}$
$b_{np} \leftarrow -0.5$
$zml_p \leftarrow 1$
3
$APLB \leftarrow [a_{np} +$
$(\ell n\ z1) b_{np}]\ zml_p$
B


B
Is
APLB ← 10⁻⁵
yes
no
Is
q > 0
yes
no
Go to 5
q ← 0
q ← q+1
Go to 4
4
Sum ← sum+APLB
p ← p+1
zml_p ← (z-1)zml_p-1
C


C
$a_{np} \leftarrow a_{np-1} \frac{n^2-0.25-p(p-1)}{2p^2} - b_{np-1} \frac{2(n^2-0.25)+p}{2p^3}$
$b_{np} \leftarrow b_{np-1} \frac{n^2-0.25-p(p-1)}{2p^2}$
Go to 3
5
q ← 0
Is
n - n VAR = 0
yes
Q^*_{12} ← sum
n VAR ← n VAR +1
Go to 1
no
D



*Quantities in A_Γ, $A_\Gamma{}'$, B_Γ Program, pertinent to the flow field of the ducted propeller.

CHAPTER FOUR

TABULATIONS

This chapter contains tables of the coefficients a_{np} and b_{np} for the small argument expression of $Q_{n-\frac{1}{2}}(z)$, see Eq. 1.12; a comparison of Riegels' Coefficients with the present computations, see Eq. 1.25; and finally, the tabulation of $Q_{n-\frac{1}{2}}(z)$ for $1 \leq n \leq 11$ arranged in increasing order. The accuracy of the tabulated values is ± 0.00001. For the lower values of order n, the decay of $Q_{n-\frac{1}{2}}(z)$ with increasing argument became less than the criterion of accuracy used. To avoid repeated values of the function only those arguments for which the functions showed a greater change are printed. Between two successive arguments, the function may be assumed stationary. The starting argument for such a behavior is marked with an asterisk (*).

TABLE 4.1

COEFFICIENTS a_{np} FROM THE SERIES
EXPANSION, Eq. (1.12)

$1 \leq n \leq 11 \ , \ 0 \leq p \leq 9$

$$a_{np} \doteq A_p \times 10^r$$

n	A_0	r	A_1	r	A_2	r	A_3	r	A_4	r
1	-2.6713200	-1	5.2482550	-1	-4.0988359	-2	9.5135320	-3	-2.9774365	-3
2	-9.3379867	-1	3.7412750	-1	6.3848104	-1	-3.9933828	-2	7.9923170	-3
3	-1.3337986	0	-1.2103689	0	1.6447669	0	9.5196744	-1	-4.9319958	-2
4	-1.6195128	0	-4.6286630	0	2.8862600	-1	4.4800694	0	1.5416869	0
5	-1.8417350	0	-1.0166471	1	-8.9948860	0	7.7372610	0	1.0743704	1
6	-2.0235531	0	-1.8046012	1	-3.5074972	1	-5.9520900	0	3.2340385	1
7	-2.1773992	0	-2.8449106	1	-9.0458440	1	-8.2289150	1	3.9626060	1
8	-2.3107325	0	-4.1529598	1	-1.9156243	2	-3.1730365	2	-1.0766607	2
9	-2.4283795	0	-5.7420822	1	-3.5894520	2	-8.8525760	2	-8.3490730	2
10	-2.5336426	0	-7.6240420	1	-6.1750598	2	-2.0735330	3	-3.1629255	3
11	-2.6288806	0	-9.8093670	1	-9.9666190	2	-4.3246418	3	-9.2188160	3

n	A_5	r	A_6	r	A_7	r	A_8	r	A_9	r
1	1.0682072	-4	-4.1387987	-4	1.6838781	-4	-7.0848231	-5	3.0545326	-5
2	-2.2670549	-3	7.5940814	-4	-2.7984938	-4	1.0960892	-4	-4.4765875	-5
3	8.5194960	-3	-2.1639072	-3	6.6626144	-4	-2.2987587	-4	8.5421695	-5
4	-6.8324340	-2	1.0352863	-2	-2.3675008	-3	6.6979767	-4	-2.1566015	-4
5	2.6126850	0	-1.0137735	-1	1.3677993	-2	-2.8393459	-3	7.4091627	-4
6	2.4340249	1	4.5575435	0	-1.5751635	-1	1.9158619	-2	-3.6388782	-3
7	1.0243045	2	5.3435553	1	8.1115530	0	-2.5302714	-1	2.8024902	-2
8	2.4407728	2	2.8700001	2	1.1502444	2	1.4652644	1	-4.1685044	-1
9	1.6639420	2	9.5252590	2	7.4959468	2	2.4432132	2	2.6772062	1
10	1.4850393	3	1.8632715	3	3.1221550	3	1.8700993	3	5.1402364	2
11	-9.0038160	3	-1.4137000	2	8.8178437	3	9.2880021	3	4.5169640	3

TABLE 4.2

COEFFICIENTS b_{np} FROM THE SERIES EXPANSION, Eq. (1.12)

$1 \leq n \leq 11 \ , \ 0 \leq p \leq 9$

$$b_{\breve{n}p} = B_p \times 10^r$$

n	B_0	r	B_1	r	B_2	r	B_3	r	B_4	r
1	-5.0000000	-1	1.8750000	-1	2.9296875	-2	-8.5449219	-3	3.0040741	-3
2	-5.0000000	-1	-9.3750000	-1	-2.0507812	-0	2.[illegible]534765	-2	-6.6089628	-3
3	-5.0000000	-1	-2.1875000	-1	-1.8457031	0	-2.81982[illegible]2	1	2.8638839	-2
4	-5.0000000	-1	-3.9375000	0	-6.7675781	0	-3.6657714	0	-4.2958258	-1
5	-5.0000000	-1	-6.1875000	0	-2.1795703	1	-1.8328857	1	-7.3029039	0
6	-5.0000000	-1	8.9375000	0	-3.7705078	1	-6.2318115	1	-4.6251725	1
7	-5.0000000	-1	-1.2187500	1	-7.1220703	1	-1.6914916	2	-1.9425723	2
8	-5.0000000	-1	-1.5937500	1	-1.2301757	2	-3.9468136	2	-6.3827376	2
9	-5.0000000	-1	-2.0187500	1	-1.9872070	2	-8.2524291	2	-1.7729828	3
10	-5.0000000	-1	-2.4937500	1	-3.0470507	2	-1.5870055	3	-4.3518666	3
11	-5.0000000	-1	-3.0187500	1	-4.4809570	2	-2.8566100	3	-9.7080105	3

n	B_5	r	B_6	r	B_7	r	B_8	r	B_9	r
1	-1.1565685	-3	4.6985595	-4	-1.9777100	-4	-8.5365999	-5	-3.7545230	-5
2	2.1479129	-3	-7.8309323	-4	3.0564608	-4	-1.2476568	-4	5.2563319	-5
3	-6.4437387	-3	1.9017978	-3	-6.4525281	-4	2.3818902	-4	-9.2996639	-5
4	3.6514519	-2	-7.2268320	-3	1.9357585	-3	-6.0870531	-4	2.1135600	-4
5	-6.9377587	-1	5.0587824	-2	-8.9044895	-3	2.1739476	-3	-6.3406805	-4
6	-1.4569293	1	-1.1635199	0	7.4204075	-2	-1.1739316	-2	2.6268530	-3
7	-1.1169790	2	-2.9087995	1	-2.0035098	0	1.1348004	-1	-1.6286487	-2
8	-5.5848954	2	-2.6179197	2	-5.8101790	1	-3.5178818	0	1.7915138	-1
9	-2.1541741	3	-1.5183935	3	-6.0038517	2	-1.1609010	2	-6.2702986	0
10	-6.9412272	3	-6.7243138	3	-3.9625420	3	-1.3543844	3	-2.3200103	2
11	-1.9561641	4	-2.4655818	4	-1.9812710	4	-1.0022445	4	-3.0160135	3

TABLE 4.3

COMPARISON OF RIEGELS' COEFFICIENTS
WITH THE Eq. (1.26)

$0 \leq n \leq 7$

	c_{n_2}		d_{n_2}	
n	Eq. 1.26	Riegels	Eq. 1.26	Riegels
0	0.500	0.500	- 0.250	- 0.250
1	- 1.500	- 1.500	1.750	1.750
2	- 7.500	- 7.500	15.750	15.750
3	- 17.500	- 17.500	44.417	44.417
4	- 31 500	- 31.500	89.350	89.350
5	- 49.500	- 49.500	151.693	151.693
6	- 71.500	- 71.500	232.334	232.334
7	- 97.500	- 97.500	332.002	332.001

TABLE 4.4

$$Q_{1/2}(z)$$

$$1.0001 \leq z \leq 1111.8889$$

$$z = 1+y^2 \quad \Delta y = 0.01$$

(x)	$Q_{1/2}(x)$	(x)	$Q_{1/2}(x)$	(x)	$Q_{1/2}(x)$
1.0001	4.33826	1.1681	0.76637	1.6561	0.31988
1.0004	3.64569	1.1764	0.74761	1.6724	0.31370
1.0009	3.24108	1.1849	0.72945	1.6889	0.30767
1.0016	2.95451	1.1936	0.71188	1.7056	0.30177
1.0025	2.73272	1.2025	0.69487	1.7225	0.29602
1.0036	2.55196	1.2116	0.67840	1.7396	0.29040
1.0049	2.39958	1.2209	0.66244	1.7569	0.28492
1.0064	2.26801	1.2304	0.64697	1.7744	0.27956
1.0081	2.15237	1.2401	0.63196	1.7921	0.27432
1.0100	2.04932	1.2500	0.61741	1.8100	0.26921
1.0121	1.95648	1.2601	0.60330	1.8281	0.26421
1.0144	1.87210	1.2704	0.58959	1.8464	0.25932
1.0169	1.79484	1.2809	0.57629	1.8649	0.25455
1.0196	1.72366	1.2916	0.56337	1.8836	0.24989
1.0225	1.65773	1.3025	0.55082	1.9025	0.24533
1.0256	1.59638	1.3136	0.53863	1.9216	0.24087
1.0289	1.53908	1.3249	0.52678	1.9409	0.23651
1.0324	1.48536	1.3364	0.51526	1.9604	0.23225
1.0361	1.43484	1.3481	0.50406	1.9801	0.22809
1.0400	1.38723	1.3600	0.49316	2.0000	0.22401
1.0441	1.34222	1.3721	0.48257	2.0201	0.22003
1.0484	1.29958	1.3844	0.47225	2.0404	0.21613
1.0529	1.25911	1.3969	0.46222	2.0609	0.21232
1.0576	1.22063	1.4096	0.45245	2.0816	0.20859
1.0625	1.18399	1.4225	0.44294	2.1025	0.20494
1.0676	1.14903	1.4356	0.43368	2.1236	0.20137
1.0729	1.11564	1.4489	0.42466	2.1449	0.19788
1.0784	1.08371	1.4624	0.41587	2.1664	0.19446
1.0841	1.05313	1.4761	0.40732	2.1881	0.19111
1.0900	1.02383	1.4900	0.39898	2.2100	0.18784
1.0961	0.99571	1.5041	0.39085	2.2321	0.18463
1.1024	0.96870	1.5184	0.38292	2.2544	0.18150
1.1089	0.94274	1.5329	0.37520	2.2769	0.17842
1.1156	0.91777	1.5476	0.36767	2.2996	0.17542
1.1225	0.89372	1.5625	0.36032	2.3225	0.17247
1.1296	0.87056	1.5776	0.35316	2.3456	0.16959
1.1369	0.84822	1.5929	0.34617	2.3689	0.16676
1.1444	0.82667	1.6084	0.33936	2.3924	0.16399
1.1521	0.80588	1.6241	0.33271	2.4161	0.16129
1.1600	0.78579	1.6400	0.32621	2.4400	0.15863

z	$Q_{1/2}(z)$	z	$Q_{1/2}(z)$	z	$Q_{1/2}(z)$
2.4641	0.15603	3.5921	0.08470	5.0401	0.05001
2.4884	0.15348	3.6244	0.08351	5.0804	0.04940
2.5129	0.15099	3.6569	0.08234	5.1209	0.04880
2.5376	0.14854	3.6896	0.08120	5.1616	0.04821
2.5625	0.14614	3.7225	0.08007	5.2025	0.04763
2.5876	0.14379	3.7556	0.07897	5.2436	0.04706
2.6129	0.14149	3.7889	0.07788	5.2849	0.04650
2.6384	0.13924	3.8224	0.07681	5.3264	0.04594
2.6641	0.13703	3.8561	0.07576	5.3681	0.04539
2.6900	0.13486	3.8900	0.07473	5.4100	0.04485
2.7161	0.13273	3.9241	0.07372	5.4521	0.04432
2.7424	0.13065	3.9584	0.07272	5.4944	0.04380
2.7689	0.12860	3.9929	0.07174	5.5369	0.04329
2.7956	0.12660	4.0276	0.07078	5.5796	0.04278
2.8225	0.12464	4.0625	0.06983	5.6225	0.04228
2.8496	0.12271	4.0976	0.06890	5.6656	0.04179
2.8769	0.12082	4.1329	0.06799	5.7089	0.04131
2.9044	0.11897	4.1684	0.06709	5.7524	0.04083
2.9321	0.11715	4.2041	0.06620	5.7961	0.04036
2.9600	0.11537	4.2400	0.06533	5.8400	0.03990
2.9881	0.11362	4.2761	0.06448	5.8841	0.03944
3.0164	0.11190	4.3124	0.06364	5.9284	0.03900
3.0449	0.11022	4.3489	0.06281	5.9729	0.03855
3.0736	0.10856	4.3856	0.06200	6.0176	0.03812
3.1025	0.10694	4.4225	0.06120	6.0625	0.03769
3.1316	0.10535	4.4596	0.06041	6.1076	0.03726
3.1609	0.10379	4.4969	0.05963	6.1529	0.03684
3.1904	0.10226	4.5344	0.05887	6.1984	0.03643
3.2201	0.10075	4.5721	0.05812	6.2441	0.03603
3.2500	0.09928	4.6100	0.05739	6.2900	0.03563
3.2801	0.09783	4.6481	0.05666	6.3361	0.03523
3.3104	0.09640	4.6864	0.05595	6.3824	0.03484
3.3409	0.09501	4.7249	0.05524	6.4289	0.03446
3.3716	0.09363	4.7636	0.05455	6.4756	0.03408
3.4025	0.09229	4.8025	0.05387	6.5225	0.03371
3.4336	0.09096	4.8416	0.05320	6.5696	0.03334
3.4649	0.08967	4.8809	0.05254	6.6169	0.03298
3.4964	0.08839	4.9204	0.05190	6.6644	0.03263
3.5281	0.08714	4.9601	0.05126	6.7121	0.03227
3.5600	0.08591	5.0000	0.05063	6.7600	0.03193

z	$Q_{1/2}(z)$	z	$Q_{1/2}(z)$	z	$Q_{1/2}(z)$
6.8081	0.03158	8.8961	0.02105	11.3041	0.01467
6.8564	0.03124	8.9524	0.02085	11.3684	0.01454
6.9049	0.03091	9.0089	0.02066	11.4329	0.01442
6.9536	0.03058	9.0656	0.02046	11.4976	0.01430
7.0025	0.03026	9.1225	0.02027	11.5625	0.01417
7.0516	0.02994	9.1796	0.02008	11.6276	0.01406
7.1009	0.02962	9.2369	0.01989	11.6929	0.01394
7.1504	0.02931	9.2944	0.01970	11.7584	0.01382
7.2001	0.02901	9.3521	0.01952	11.8241	0.01370
7.2500	0.02870	9.4100	0.01934	11.8900	0.01359
7.3001	0.02841	9.4681	0.01916	11.9561	0.01348
7.3504	0.02811	9.5264	0.01898	12.0224	0.01337
7.4009	0.02782	9.5849	0.01881	12.0889	0.01325
7.4516	0.02753	9.6436	0.01864	12.1556	0.01315
7.5025	0.02725	9.7025	0.01847	12.2225	0.01304
7.5536	0.02697	9.7616	0.01830	12.2896	0.01293
7.6049	0.02670	9.8209	0.01813	12.3569	0.01282
7.6564	0.02642	9.8804	0.01797	12.4244	0.01272
7.7081	0.02616	9.9401	0.01780	12.4921	0.01262
7.7600	0.02589	10.0000	0.01764	12.5600	0.01251
7.8121	0.02563	10.0601	0.01748	12.6281	0.01241
7.8644	0.02537	10.1204	0.01733	12.6964	0.01231
7.9169	0.02512	10.1809	0.01717	12.7649	0.01221
7.9696	0.02487	10.2416	0.01702	12.8336	0.01211
8.0225	0.02462	10.3025	0.01687	12.9025	0.01202
8.0756	0.02437	10.3636	0.01672	12.9716	0.01192
8.1289	0.02413	10.4249	0.01657	13.0409	0.01183
8.1824	0.02389	10.4864	0.01642	13.1104	0.01173
8.2361	0.02366	10.5481	0.01628	13.1801	0.01164
8.2900	0.02343	10.6100	0.01614	13.2500	0.01155
8.3441	0.02320	10.6721	0.01599	13.3201	0.01145
8.3984	0.02297	10.7344	0.01585	13.3904	0.01136
8.4529	0.02275	10.7969	0.01572	13.4609	0.01127
8.5076	0.02253	10.8596	0.01558	13.5316	0.01119
8.5625	0.02231	10.9225	0.01544	13.6025	0.01110
8.6176	0.02209	10.9856	0.01531	13.6736	0.01101
8.6729	0.02188	11.0489	0.01518	13.7449	0.01093
8.7284	0.02167	11.1124	0.01505	13.8164	0.01084
8.7841	0.02146	11.1761	0.01492	13.8881	0.01076
8.8400	0.02126	11.2400	0.01479	13.9600	0.01067

z	$Q_{1/2}(z)$	z	$Q_{1/2}(z)$	z	$Q_{1/2}(z)$
14.0321	0.01059	17.0801	0.00788	20.4481	0.00601
14.1044	0.01051	17.1604	0.00782	20.5364	0.00597
14.1769	0.01043	17.2409	0.00777	20.6249	0.00594
14.2496	0.01035	17.3216	0.00772	20.7136	0.00590
14.3225	0.01027	17.4025	0.00766	20.8025	0.00586
14.3956	0.01019	17.4836	0.00761	20.8916	0.00582
14.4689	0.01011	17.5649	0.00756	20.9809	0.00578
14.5424	0.01004	17.6464	0.00750	21.0704	0.00575
14.6161	0.00996	17.7281	0.00745	21.1601	0.00571
14.6900	0.00988	17.8100	0.00740	21.2500	0.00568
14.7641	0.00981	17.8921	0.00735	21.3401	0.00564
14.8384	0.00974	17.9744	0.00730	21.4304	0.00560
14.9129	0.00966	18.0569	0.00725	21.5209	0.00557
14.9876	0.00959	18.1396	0.00720	21.6116	0.00553
15.0625	0.00952	18.2225	0.00715	21.7025	0.00550
15.1376	0.00945	18.3056	0.00710	21.7936	0.00546
15.2129	0.00938	18.3889	0.00705	21.8849	0.00543
15.2884	0.00931	18.4724	0.00700	21.9764	0.00540
15.3641	0.00924	18.5561	0.00696	22.0681	0.00536
15.4400	0.00917	18.6400	0.00691	22.1600	0.00533
15.5161	0.00910	18.7241	0.00686	22.2521	0.00530
15.5924	0.00904	18.8084	0.00682	22.3444	0.00526
15.6689	0.00897	18.8929	0.00677	22.4369	0.00523
15.7456	0.00890	18.9776	0.00673	22.5296	0.00520
15.8225	0.00884	19.0625	0.00668	22.6225	0.00517
15.8996	0.00878	19.1476	0.00664	22.7156	0.00513
15.9769	0.00871	19.2329	0.00659	22.8089	0.00510
16.0544	0.00865	19.3184	0.00655	22.9024	0.00507
16.1321	0.00859	19.4041	0.00651	22.9961	0.00504
16.2100	0.00852	19.4900	0.00646	23.0900	0.00501
16.2881	0.00846	19.5761	0.00642	23.1841	0.00498
16.3664	0.00840	19.6624	0.00638	23.2784	0.00495
16.4449	0.00834	19.7489	0.00634	23.3729	0.00492
16.5236	0.00828	19.8356	0.00629	23.4676	0.00489
16.6025	0.00822	19.9225	0.00625	23.5625	0.00486
16.6816	0.00816	20.0096	0.00621	23.6576	0.00483
16.7609	0.00811	20.0969	0.00617	23.7529	0.00480
16.8404	0.00805	20.1844	0.00613	23.8484	0.00477
16.9201	0.00799	20.2721	0.00609	23.9441	0.00474
17.0000	0.00794	20.3600	0.00605	24.0400	0.00472

z	$Q_{1/2}(z)$	z	$Q_{1/2}(z)$	z	$Q_{1/2}(z)$
24.1361	0.00469	28.1441	0.00372	32.4721	0.00300
24.2324	0.00466	28.2484	0.00370	32.5844	0.00299
24.3289	0.00463	28.3529	0.00368	32.6969	0.00297
24.4256	0.00460	28.4576	0.00366	32.8096	0.00296
24.5225	0.00458	28.5625	0.00364	32.9225	0.00294
24.6196	0.00455	28.6676	0.00362	33.0356	0.00293
24.7169	0.00452	28.772[illegible]	0.00360	33.1489	0.00291
24.8144	0.00450	28.8784	0.00358	33.2624	0.00290
24.9121	0.00447	28.9841	0.00356	33.3761	0.00288
25.0100	0.00444	29.0900	0.00354	33.4900	0.00287
25.1081	0.00442	29.1961	0.00352	33.6041	0.00285
25.2064	0.00439	29.3024	0.00350	33.7184	0.00284
25.3049	0.00437	29.4089	0.00348	33.8329	0.00282
25.4036	0.00434	29.5156	0.00346	33.9476	0.00281
25.5025	0.00432	29.6225	0.00345	34.0625	0.00279
25.6016	0.00429	29.7296	0.00343	34.1776	0.00278
25.7009	0.00427	29.8369	0.00341	34.2929	0.00277
25.8004	0.00424	29.9444	0.00339	34.4084	0.00275
25.9001	0.00422	30.0521	0.00337	34.5241	0.00274
26.0000	0.00419	30.1600	0.00335	34.6400	0.00272
26.1001	0.00417	30.2681	0.00334	34.7561	0.00271
26.2004	0.00414	30.3764	0.00332	34.8724	0.00270
26.3009	0.00412	30.4849	0.00330	34.9889	0.00268
26.4016	0.00410	30.5936	0.00328	35.1056	0.00267
26.5025	0.00407	30.7025	0.00327	35.2225	0.00266
26.6036	0.00405	30.8116	0.00325	35.3396	0.00264
26.7049	0.00403	30.9209	0.00323	35.4569	0.00263
26.8046	0.00400	31.0304	0.00321	35.5744	0.00262
26.9081	0.00398	31.1401	0.00320	35.6921	0.00261
27.0100	0.00396	31.2500	0.00318	35.8100	0.00259
27.1121	0.00394	31.3601	0.00316	35.9281	0.00258
27.2144	0.00391	31.4704	0.00315	36.0464	0.00257
27.3169	0.00389	31.5809	0.00313	36.1649	0.00255
27.4196	0.00387	31.6916	0.00311	36.2836	0.00254
27.5225	0.00385	31.8025	0.00310	36.4025	0.00253
27.6256	0.00383	31.9136	0.00308	36.5216	0.00252
27.7289	0.00381	32.0249	0.00307	36.6409	0.00250
27.8324	0.00378	32.1364	0.00305	36.7604	0.00249
27.9361	0.00376	32.2481	0.00303	36.8801	0.00248
28.0400	0.00374	32.3600	0.00302	37.0000	0.00247

z	$Q_{1/2}(z)$	z	$Q_{1/2}(z)$	z	$Q_{1/2}(z)$
37.1201	0.00246	42.0881	0.00203	47.3761	0.00170
37.2404	0.00244	42.2164	0.00203	47.5124	0.00170
37.3609	0.00243	42.3449	0.00202	47.6489	0.00169
37.4816	0.00242	42.4736	0.00201	47.7856	0.00168
37.6025	0.00241	42.6025	0.00200	47.9225	0.00167
37.7236	0.00240	42.7316	0.00199	48.0596	0.00167
37.8449	0.00239	42.8609	0.00198	48.1969	0.00166
37.9664	0.00237	42.9904	0.00197	48.3344	0.00165
38.0881	0.00236	43.1201	0.00196	48.4721	0.00165
38.2100	0.00235	43.2500	0.00195	48.6100	0.00164
38.3321	0.00234	43.3801	0.00194	48.7481	0.00163
38.4544	0.00233	43.5104	0.00194	48.8864	0.00162
38.5769	0.00232	43.6409	0.00193	49.0249	0.00162
38.6996	0.00231	43.7716	0.00192	49.1636	0.00161
38.8225	0.00230	43.9025	0.00191	49.3025	0.00160
38.9456	0.00229	44.0336	0.00190	49.4416	0.00160
39.0689	0.00227	44.1649	0.00189	49.5809	0.00159
39.1924	0.00226	44.2964	0.00188	49.7204	0.00158
39.3161	0.00225	44.4281	0.00188	49.8601	0.00158
39.4400	0.00224	44.5600	0.00187	50.0000	0.00157
39.5641	0.00223	44.6921	0.00186	50.1401	0.00156
39.6884	0.00222	44.8244	0.00185	50.2804	0.00156
39.8129	0.00221	44.9569	0.00184	50.4209	0.00155
39.9376	0.00220	45.0896	0.00183	50.5616	0.00154
40.0625	0.00219	45.2225	0.00183	50.7025	0.00154
40.1876	0.00218	45.3556	0.00182	50.8436	0.00153
40.3129	0.00217	45.4889	0.00181	50.9849	0.00153
40.4384	0.00216	45.6224	0.00180	51.1264	0.00152
40.5641	0.00215	45.7561	0.00179	51.2681	0.00151
40.6900	0.00214	45.8900	0.00179	51.4100	0.00151
40.8161	0.00213	46.0241	0.00178	51.5521	0.00150
40.9424	0.00212	46.1584	0.00177	51.6944	0.00149
41.0689	0.00211	46.2929	0.00176	51.8369	0.00149
41.1956	0.00210	46.4276	0.00176	51.9796	0.00148
41.3225	0.00209	46.5625	0.00175	52.1225	0.00148
41.4496	0.00208	46.6976	0.00174	52.2656	0.00147
41.5769	0.00207	46.8329	0.00173	52.4089	0.00146
41.7044	0.00206	46.9684	0.00173	52.5524	0.00146
41.8321	0.00205	47.1041	0.00172	52.6961	0.00145
41.9600	0.00204	47.2400	0.00171	52.8400	0.00145

z	$Q_{1/2}(z)$	z	$Q_{1/2}(z)$	z	$Q_{1/2}(z)$
52.9841	0.00144	58.9121	0.00123	65.1601	0.00106
53.1284	0.00143	59.0644	0.00122	65.3204	0.00105
53.2729	0.00143	59.2169	0.00122	65.4809	0.00105
53.4176	0.00142	59.3696	0.00121	65.6416	0.00104
53.5625	0.00142	59.5225	0.00121	65.8025	0.00104
53.7076	0.00141	59.6756	0.00120	65.9636	0.00104
53.8529	0.00141	59.8289	0.00120	66.1249	0.00103
53.9984	0.00140	59.9824	0.00120	66.2864	0.00103
54.1441	0.00139	60.1361	0.00119	66.4481	0.00103
54.2900	0.00139	60.2900	0.00119	66.6100	0.00102
54.4361	0.00138	60.4441	0.00118	66.7721	0.00102
54.5824	0.00138	60.5984	0.00118	66.9344	0.00101
54.7289	0.00137	60.7529	0.00117	67.0969	0.00101
54.8756	0.00137	60.9076	0.00117	67.2596	0.00101
55.0225	0.00136	61.0625	0.00116	67.4225	0.00100
55.1696	0.00136	61.2176	0.00116	67.5856	0.00100
55.3169	0.00135	61.3729	0.00116	67.7489	0.00100
55.4644	0.00134	61.5284	0.00115	67.9124	0.00099
55.6121	0.00134	61.6841	0.00115	68.0761	0.00099
55.7600	0.00133	61.8400	0.00114	68.2400	0.00099
55.9081	0.00133	61.9961	0.00114	68.4041	0.00098
56.0564	0.00132	62.1524	0.00113	68.5684	0.00098
56.2049	0.00132	62.3089	0.00113	68.7329	0.00097
56.3536	0.00131	62.4656	0.00113	68.8976	0.00097
56.5025	0.00131	62.6225	0.00112	69.0625	0.00097
56.6516	0.00130	62.7796	0.00112	69.2276	0.00096
56.8009	0.00130	62.9369	0.00111	69.3929	0.00096
56.9504	0.00129	63.0944	0.00111	69.5584	0.00096
57.1001	0.00129	63.2521	0.00110	69.7241	0.00095
57.2500	0.00128	63.4100	0.00110	69.8900	0.00095
57.4001	0.00128	63.5681	0.00110	70.0561	0.00095
57.5504	0.00127	63.7264	0.00109	70.2224	0.00094
57.7009	0.00127	63.8849	0.00109	70.3889	0.00094
57.8516	0.00126	64.0436	0.00108	70.5556	0.00094
58.0025	0.00126	64.2025	0.00108	70.7225	0.00093
58.1536	0.00125	64.3616	0.00108	70.8896	0.00093
58.3049	0.00125	64.5209	0.00107	71.0569	0.00093
58.4564	0.00124	64.6804	0.00107	71.2244	0.00092
58.6081	0.00124	64.8401	0.00106	71.3921	0.00092
58.7600	0.00123	65.0000	0.00106	71.5600	0.00092

z	$Q_{1/2}(z)$	z	$Q_{1/2}(z)$	z	$Q_{1/2}(z)$
71.7281	0.00091	78.6161	0.00080	90.6809	0.00064
71.8964	0.00091	78.7924	0.00079	91.6304	0.00063
72.0649	0.00091	78.9689	0.00079	92.5849	0.00062
72.2336	0.00090	79.1456	0.00079	93.5444	0.00061
72.4025	0.00090	79.3225	0.00079	94.5089	0.00060
72.5716	0.00090	79.4996	0.00078	95.6729	0.00059
72.7409	0.00090	79.6769	0.00078	96.6484	0.00058
72.9104	0.00089	79.8544	0.00078	97.8256	0.00057
73.0801	0.00089	80.0321	0.00078	99.0100	0.00056
73.2500	0.00089	80.2100	0.00077	100.2016	0.00055
73.4201	0.00088	80.3881	0.00077	101.4004	0.00054
73.5904	0.00088	80.5664	0.00077	102.6064	0.00053
73.7609	0.00088	80.7449	0.00077	103.8196	0.00052
73.9316	0.00087	80.9236	0.00076	105.2441	0.00051
74.1025	0.00087	81.1025	0.00076	106.6784	0.00050
74.2736	0.00087	81.2816	0.00076	108.1225	0.00049
74.4449	0.00086	81.4609	0.00076	109.5764	0.00048
74.6164	0.00086	81.6404	0.00075	111.0401	0.00047
74.7881	0.00086	81.8201	0.00075	112.7249	0.00046
74.9600	0.00086	82.0000	0.00075	114.2096	0.00045
75.1321	0.00085	82.1801	0.00075	115.9184	0.00044
75.3044	0.00085	82.3604	0.00074	117.8561	0.00043
75.4769	0.00085	82.5409	0.00074	119.5921	0.00042
75.6496	0.00084	82.7216	0.00074	121.5604	0.00041
75.8225	0.00084	82.9025	0.00074	123.5449	0.00040
75.9956	0.00084	83.0836	0.00073	125.5456	0.00039
76.1689	0.00084	83.2649	0.00073	127.7876	0.00038
76.3424	0.00083	83.4464	0.00073	130.0496	0.00037
76.5161	0.00083	83.6281	0.00073	132.3316	0.00036
76.6900	0.00083	83.8100	0.00072	134.8649	0.00035
76.8641	0.00082	83.9921	0.00072	137.1889	0.00034
77.0384	0.00082	84.1744	0.00072	140.2400	0.00033
77.2129	0.00082	84.3569	0.00072	143.0864	0.00032
77.3876	0.00082	84.5396*	0.00071	145.9616	0.00031
77.5625	0.00081	85.4561	0.00070	149.1089	0.00030
77.7376	0.00081	86.1929	0.00069	152.5361	0.00029
77.9129	0.00081	87.1184	0.00068	156.0025	0.00028
78.0884	0.00080	87.8624	0.00067	160.0121	0.00027
78.2641	0.00080	88.7969	0.00066	163.8176	0.00026
78.4400	0.00080	89.7364	0.00065	168.1849	0.00025

z	$Q_{1/2}(z)$
172.6100	0.00024
177.6241	0.00023
182.7104	0.00022
188.4161	0.00021
194.4881	0.00020
200.9396	0.00019
208.3600	0.00018
216.2089	0.00017
224.8016	0.00016
234.1729	0.00015
244.9844	0.00014
257.0000	0.00013
270.2881	0.00012
285.9344	0.00011
303.7600	0.00010
324.6401	0.00009
349.5689	0.00008
800.8090	0.00007
417.9764	0.00006
467.5600	0.00005
534.1481	0.00004
631.5121	0.00003
790.6100	0.00002
1111.2224	0.00001

TABLE 4.5

$$Q_{3/2}(z)$$

$$1.0001 \leq z \leq 45.8890$$

$$z = 1+y^2 \quad \Delta y = 0.01$$

z	$Q_{3/2}(z)$	z	$Q_{3/2}(z)$	z	$Q_{3/2}(z)$
1.0001	3.67227	1.1681	0.32969	1.6561	0.08101
1.0004	2.98131	1.1764	0.31711	1.6724	0.07847
1.0009	2.57901	1.1849	0.30509	1.6889	0.07601
1.0016	2.29534	1.1936	0.29360	1.7056	0.07365
1.0025	2.07692	1.2025	0.28262	1.7225	0.07136
1.0036	1.89997	1.2116	0.27211	1.7396	0.06916
1.0049	1.75177	1.2209	0.26204	1.7569	0.06703
1.0064	1.62470	1.2304	0.25241	1.7744	0.06498
1.0081	1.51385	1.2401	0.24319	1.7921	0.06299
1.0100	1.41586	1.2500	0.23435	1.8100	0.06108
1.0121	1.32831	1.2601	0.22587	1.8281	0.05923
1.0144	1.24941	1.2704	0.21775	1.8464	0.05744
1.0169	1.17782	1.2809	0.20996	1.8649	0.05572
1.0196	1.11246	1.2916	0.20249	1.8836	0.05405
1.0225	1.05249	1.3025	0.19532	1.9025	0.05244
1.0256	0.99723	1.3136	0.18844	1.9216	0.05088
1.0289	0.94613	1.3249	0.18184	1.9409	0.04938
1.0324	0.89870	1.3364	0.17550	1.9604	0.04792
1.0361	0.85456	1.3481	0.16941	1.9801	0.04652
1.0400	0.81338	1.3600	0.16356	2.0000	0.04516
1.0441	0.77487	1.3721	0.15793	2.0201	0.04384
1.0484	0.73878	1.3844	0.15253	2.0404	0.04257
1.0529	0.70490	1.3969	0.14733	2.0609	0.04134
1.0576	0.67304	1.4096	0.14234	2.0816	0.04015
1.0625	0.64302	1.4225	0.13753	2.1025	0.03900
1.0676	0.61472	1.4356	0.13291	2.1236	0.03789
1.0729	0.58798	1.4489	0.12847	2.1449	0.03681
1.0784	0.56271	1.4624	0.12419	2.1664	0.03577
1.0841	0.53878	1.4761	0.12007	2.1881	0.03476
1.0900	0.51612	1.4900	0.11611	2.2100	0.03378
1.0961	0.49461	1.5041	0.11229	2.2321	0.03283
1.1024	0.47420	1.5184	0.10861	2.2544	0.03192
1.1089	0.45482	1.5329	0.10507	2.2769	0.03103
1.1156	0.43639	1.5476	0.10166	2.2996	0.03017
1.1225	0.41885	1.5625	0.09838	2.3225	0.02934
1.1296	0.40216	1.5776	0.09521	2.3456	0.02853
1.1369	0.38626	1.5929	0.09216	2.3689	0.02775
1.1444	0.37110	1.6084	0.08922	2.3924	0.02699
1.1521	0.35665	1.6241	0.08638	2.4161	0.02626
1.1600	0.34286	1.6400	0.08365	2.4400	0.02555

z	$\Omega_{3/2}(z)$	z	$\Omega_{3/2}(z)$	z	$\Omega_{3/2}(z)$
2.4641	0.02486	3.5921	0.00903	5.0401	0.00376
2.4884	0.02419	3.6244	0.00882	5.0804	0.00368
2.5129	0.02354	3.6569	0.00861	5.1209	0.00361
2.5376	0.02292	3.6896	0.00842	5.1616	0.00353
2.5625	0.02231	3.7225	0.00822	5.2025	0.00346
2.5876	0.02172	3.7556	0.00803	5.2436	0.00339
2.6129	0.02114	3.7889	0.00785	5.2849	0.00333
2.6384	0.02059	3.8224	0.00767	5.3264	0.00326
2.6641	0.02005	3.8561	0.00750	5.3681	0.00320
2.6900	0.01953	3.8900	0.00733	5.4100	0.00313
2.7161	0.01902	3.9241	0.00717	5.4521	0.00307
2.7424	0.01853	3.9584	0.00701	5.4944	0.00301
2.7689	0.01805	3.9929	0.00685	5.5369	0.00296
2.7956	0.01759	4.0276	0.00670	5.5796	0.00290
2.8225	0.01714	4.0625	0.00655	5.6225	0.00284
2.8496	0.01670	4.0976	0.00641	5.6656	0.00279
2.8769	0.01628	4.1329	0.00626	5.7089	0.00273
2.9044	0.01586	4.1684	0.00613	5.7524	0.00268
2.9321	0.01546	4.2041	0.00599	5.7961	0.00263
2.9600	0.01508	4.2400	0.00586	5.8400	0.00258
2.9881	0.01470	4.2761	0.00574	5.8841	0.00253
3.0164	0.01433	4.3124	0.00561	5.9284	0.00248
3.0449	0.01398	4.3489	0.00549	5.9729	0.00244
3.0736	0.01363	4.3856	0.00537	6.0176	0.00239
3.1025	0.01329	4.4225	0.00526	6.0625	0.00235
3.1316	0.01297	4.4596	0.00515	6.1076	0.00230
3.1609	0.01265	4.4969	0.00504	6.1529	0.00226
3.1904	0.01234	4.5344	0.00493	6.1984	0.00222
3.2201	0.01204	4.5721	0.00483	6.2441	0.00217
3.2500	0.01175	4.6100	0.00473	6.2900	0.00213
3.2801	0.01147	4.6481	0.00463	6.3361	0.00210
3.3104	0.01119	4.6864	0.00453	6.3824	0.00206
3.3409	0.01092	4.7249	0.00444	6.4289	0.[illegible]202
3.3716	0.01066	4.7636	0.00434	6.4756	0.00198
3.4025	0.01041	4.8025	0.00425	6.5225	0.00195
3.4336	0.01016	4.8416	0.00417	6.5696	0.00191
3.4649	0.00992	4.8809	0.00408	6.6169	0.00188
3.4964	0.00969	4.9204	0.00400	6.6644	0.00184
3.5281	0.00946	4.9601	0.00392	6.7121	0.00181
3.5600	0.00924	5.0000	0.00384	6.7600	0.00178

z	$Q_{3/2}(z)$	z	$Q_{3/2}(z)$	z	$Q_{3/2}(z)$
6.8081	0.00175	8.8961	0.00089	11.3041	0.00049
6.8564	0.00172	8.9524	0.00087	11.3684	0.00048
6.9049	0.00169	9.0089	0.00086	11.4329	0.00047
6.9536	0.00166	9.0656	0.00085	11.4976	0.00047
7.0025	0.00163	9.1225	0.00083	11.5625	0.00046
7.0516	0.00160	9.1796	0.00082	11.6276	0.00045
7.1009	0.00157	9.2369	0.00081	11.6929	0.00045
7.1504	0.00154	9.2944	0.00080	11.7584	0.00044
7.2001	0.00152	9.3521	0.00078	11.8241	0.00043
7.2500	0.00149	9.4100	0.00077	11.8900	0.00043
7.3001	0.00147	9.4681	0.00076	11.9561	0.00042
7.3504	0.00144	9.5264	0.00075	12.0224	0.00042
7.4009	0.00142	9.5849	0.00074	12.0889	0.00041
7.4516	0.00139	9.6436	0.00073	12.1556	0.00040
7.5025	0.00137	9.7025	0.00071	12.2225	0.00040
7.5536	0.00134	9.7616	0.00070	12.2896	0.00039
7.6049	0.00132	9.8209	0.00069	12.3569	0.00039
7.6564	0.00130	9.8804	0.00068	12.4244	0.00038
7.7081	0.00128	9.9401	0.00067	12.4921	0.00038
7.7600	0.00126	10.0000	0.00066	12.5600	0.00037
7.8121	0.00123	10.0601	0.00065	12.6281	0.00037
7.8644	0.00121	10.1204	0.00064	12.6964	0.00036
7.9169	0.00119	10.1809	0.00063	12.7649	0.00036
7.9696	0.00117	10.2416	0.00062	12.8336	0.00035
8.0225	0.00115	10.3025	0.00061	12.9025	0.00035
8.0756	0.00114	10.3636	0.00061	12.9716	0.00034
8.1289	0.00112	10.4249	0.00060	13.0409	0.00034
8.1824	0.00110	10.4864	0.00059	13.1104	0.00033
8.2361	0.00108	10.5481	0.00058	13.1801	0.00033
8.2900	0.00106	10.6100	0.00057	13.2500	0.00033
8.3441	0.00104	10.6721	0.00056	13.3201	0.00032
8.3984	0.00103	10.7344	0.00055	13.3904	0.00032
8.4529	0.00101	10.7969	0.00055	13.4609	0.00031
8.5076	0.00099	10.8596	0.00054	13.5316	0.00031
8.5625	0.00098	10.9225	0.00053	13.6025	0.00031
8.6176	0.00096	10.9856	0.00052	13.6736	0.00030
8.6729	0.00095	10.0489	0.00052	13.7449	0.00030
8.7284	0.00093	11.1124	0.00051	13.8164	0.00029
8.7841	0.00091	11.1761	0.00050	13.8881	0.00029
8.8400	0.00090	11.2400	0.00049	13.9600	0.00029

z	$Q_{3/2}(z)$	z	$Q_{3/2}(z)$
14.0321	0.00028	21.7025	0.00009
14.1044	0.00028	22.6225	0.00008
14.1769	0.00028	23.8484	0.00007
14.2496	0.00027	25.2064	0.00006
14.3225	0.00027	27.0100	0.00005
14.3956	0.00027	29.3024	0.00004
14.4689	0.00026	32.3600	0.00003
14.5424	0.00026	37.0000	0.00002
14.6161	0.00026	45.4889	0.00001
14.6900	0.00025		
14.7641	0.00025		
14.8384	0.00025		
14.9129	0.00024		
14.9876	0.00024		
15.0625	0.00024		
15.1376	0.00023		
15.2129	0.00023		
15.2884	0.00023		
15.3641	0.00023		
15.4400	0.00022		
15.5161	0.00022		
15.5924	0.00022		
15.6689	0.00021		
15.7456	0.00021		
15.6225	0.00021		
15.8996	0.00021		
15.9769	0.00020		
16.0544	0.00020		
16.1321	0.00020		
16.2100	0.00020		
16.2881*	0.00019		
16.6816	0.00018		
17.0000	0.00017		
17.4025	0.00016		
17.8921	0.00015		
18.3889	0.00014		
18.8929	0.00013		
19.4900	0.00012		
20.1844	0.00011		
20.8916	0.00010		

TABLE 4.6

$Q_{5/2}(z)$

$1.0001 \leq z \leq 11.7584$

$z = 1+y^2 \quad \Delta y = 0.01$

z	$Q_{5/2}(z)$	z	$Q_{5/2}(z)$	z	$Q_{5/2}(z)$
1.0001	3.27327	1.1681	0.1563[illegible]	1.6561	0.02275
1.0004	2.58459	1.1764	0.14834	1.6724	0.02176
1.0[illegible]09	2.18549	1.1849	0.14076	1.6889	0.02083
1.0016	1.90571	1.1936	0.13361	1.7056	0.01993
1.0025	1.69175	1.2025	0.12686	1.7225	0.01908
1.0036	1.51972	1.2116	[illegible].12047	1.7396	0.01827
1.0049	1.37681	1.2209	0.11444	1.7569	0.01749
1.0064	1.25535	1.2304	0.10874	1.7744	0.01675
1.0081	1.15037	1.2401	0.10335	1.7921	0.01605
1.0100	1.05844	1.2500	0.09826	1.8100	0.01537
1.0121	0.97712	1.2601	0.09344	1.8281	0.01473
1.0144	0.90458	1.2704	0.08887	1.8464	0.01412
1.0169	0.83945	1.2809	0.08455	1.8649	0.01353
1.0196	0.78062	1.2916	0.08046	1.8836	0.01297
1.0225	0.72724	1.3025	0.07658	1.9025	0.01244
1.0256	0.67859	1.3136	0.07290	1.9216	0.01193
1.0289	0.63410	1.3249	0.06942	1.9409	0.01144
1.0324	0.59329	1.3364	0.06612	1.9604	0.01097
1.0361	0.55575	1.3481	0.06299	1.9801	0.01053
1.0400	0.52113	1.3600	0.06001	2.0000	0.01010
1.0441	0.48914	1.3721	0.05719	2.0201	0.00970
1.0484	0.45951	1.3844	0.05452	[illegible].0404	0.00931
1.0529	0.43203	1.3969	0.05198	2.0609	0.00894
1.0576	0.40650	1.4096	0.04957	2.0816	0.00858
1.0625	0.38275	1.4225	0.04728	2.1025	0.00824
1.0676	0.36062	1.4356	0.04510	2.1236	0.00791
1.0729	0.33997	1.4489	0.04303	2.1449	0.00760
1.0784	0.32069	1.4624	0.04107	2.1664	0.00730
1.0841	0.30267	1.4761	0.03920	2 [illegible]881	0.00702
1.0900	0.28581	1.4900	0.03743	[illegible]100	0.00675
1.0961	0.27003	1.5041	0.03574	[illegible]321	0.00648
1.1024	0.25522	1.5184	0.03413	2.2544	0.00623
1.1089	0.24134	1.5329	0.03260	2.2769	0.00599
1.1156	0.22830	1.5476	0.03115	2.2996	0.00576
[illegible]	0.21605	1.[illegible]625	0.02[illegible]	2.3225	0.00[illegible]54
1.1296	0.20454	1.5776	0.02845	2.3456	0.00533
1.1369	0.19372	1.[illegible]929	0.02719	2.3689	0.00513
1.1444	0.18353	1.[illegible]84	0.02600	2.3924	0.00494
1.1521	0.17393	1.6241	0.02486	2.4161	0.00475
1.1600	0.16489	1.6400	0.02378	2.4400	0.00457

z	$Q_{5/2}(z)$	z	$Q_{5/2}(z)$	z	$Q_{5/2}(z)$
2.4641	0.00440	3.5921	0.00107	5.0401	0.00031
2.4884	0.00424	3.6244	0.00104	5.0804	0.00030
2.5129	0.00408	3.6569	0.00100	5.1209	0.00029
2.5376	0.00393	3.6896	0.00097	5.1616	0.00029
2.5625	0.00378	3.7225	0.00094	5.2025	0.00028
2.5876	0.00365	3.7556	0.00091	5.2436	0.00027
2.6129	0.00351	3.7889	0.00088	5.2849	0.00026
2.6384	0.00338	3.8224	0.00085	5.3264	0.00026
2.6641	0.00326	3.8561	0.00083	5.3681	0.00025
2.6900	0.00314	3.8900	0.00080	5.4100	0.00024
2.7161	0.00303	3.9241	0.00078	5.4521	0.00023
2.7424	0.00292	3.9584	0.00075	5.4944	0.00023
2.7689	0.00282	3.9929	0.00073	5.5369	0.00022
2.7956	0.00272	4.0276	0.00071	5.5796	0.00021
2.8225	0.00262	4.0625	0.00069	5.6225	0.00021
2.8496	0.00253	4.0976	0.00066	5.6656	0.00020
2.8769	0.00244	4.1329	0.00064	5.7089	0.00020
2.9044	0.00235	4.1684	0.00062	5.7524	0.00019
2.9321	0.00227	4.2041	0.00061	5.7961	0.00019
2.9600	0.00219	4.2400	0.00059	5.8400	0.00018
2.9881	0.00212	4.2761	0.00057	5.8841	0.00018
3.0164	0.00204	4.3124	0.00055	5.9284	0.00017
3.0449	0.00197	4.3489	0.00054	5.9729	0.00017
3.0736	0.00190	4.3856	0.00052	6.0176	0.00016
3.1025	0.00184	4.4225	0.00051	6.0625	0.00016
3.1316	0.00178	4.4596	0.00049	6.1076	0.00016
3.1609	0.00172	4.4969	0.00048	6.1529	0.00015
3.1904	0.00166	4.5344	0.00046	6.1984	0.00015
3.2201	0.00160	4.5721	0.00045	6.2441	0.00014
3.2500	0.00155	4.6100	0.00044	6.2900	0.00014
3.2801	0.00150	4.6481	0.00042	6.3361	0.00014
3.3104	0.00145	4.6864	0.00041	6.3824	0.00013
3.3409	0.00140	4.7249	0.00040	6.4289	0.00013
3.3716	0.00135	4.7636	0.00039	6.4756	0.00013
3.4025	0.00131	4.8025	0.00038	6.5225	0.00012
3.4336	0.00126	4.8416	0.00037	6.5696	0.00012
3.4649	0.00122	4.8809	0.00036	6.6169	0.00012
3.4964	0.00118	4.9204	0.00035	6.6644	0.00011
3.5281	0.00114	4.9601	0.00034	6.7121	0.00011
3.5600	0.00111	5.0000	0.00033	6.7600	0.00011

z	$Q_{5/2}(z)$		
6.8081	0.00010		
6.8564	0.00010		
6.9049*	0.00010		
7.2500	0.00008		
7.5025	0.00007		
7.8121	0.00006		
8.1824	0.00005		
8.6729	0.00004		
9.1796	0.00003		
10.1204	0.00002		
11.7584	0.00001		

TABLE 4.7

$$Q_{7/2}(z)$$

$$1.0001 \leq z \leq 5.421$$

$$z = 1+y^2 \quad \Delta y + 0.01$$

z	$Q_{7/2}(z)$	z	$Q_{7/2}(z)$	z	$Q_{7/2}(z)$
1.0001	2.98882	1.1681	0.07760	1.6561	0.00669
1.0004	2.30299	1.1764	0.07261	1.6724	0.00632
1.0009	1.90777	1.1849	0.06796	1.6889	0.00598
1.0016	1.63263	1.1936	0.06363	1.7056	0.00565
1.0025	1.42388	1.2025	0.05959	1.7225	0.00534
1.0036	1.25749	1.2116	0.05584	1.7396	0.00505
1.0049	1.12055	1.2209	0.05233	1.7569	0.00478
1.0064	1.00531	1.2304	0.04905	1.7744	0.00452
1.0081	0.90671	1.2401	0.04599	1.7921	0.00428
1.0100	0.82128	1.2500	0.04314	1.8100	0.00405
1.0121	0.74653	1.2601	0.04048	1.8281	0.00383
1.0144	0.68061	1.2704	0.03798	1.8464	0.00363
1.0169	0.62207	1.2809	0.03566	1.8649	0.00344
1.0196	0.56982	1.2916	0.03348	1.8836	0.00326
1.0225	0.52296	1.3025	0.03145	1.9025	0.00309
1.0256	0.48077	1.3136	0.02954	1.9216	0.00293
1.0289	0.44265	1.3249	0.02776	1.9409	0.00277
1.0324	0.40810	1.3364	0.02609	1.9604	0.00263
1.0361	0.37671	1.3481	0.02453	1.9801	0.00249
1.0400	0.34812	1.3600	0.02307	2.0000	0.00237
1.0441	0.32202	1.3721	0.02170	2.0201	0.00224
1.0484	0.29816	1.3844	0.02041	2.0404	0.00213
1.0529	0.27631	1.3969	0.01921	2.0609	0.00202
1.0576	0.25626	1.4096	0.01808	2.0816	0.00192
1.0625	0.23785	1.4225	0.01703	2.1025	0.00182
1.0676	0.22091	1.4356	0.01603	2.1236	0.00173
1.0729	0.20531	1.4489	0.01510	2.1449	0.00164
1.0784	0.19093	1.4624	0.01423	2.1664	0.00156
1.0841	0.17766	1.4761	0.01341	2.1881	0.00148
1.0900	0.16540	1.4900	0.01264	2.2100	0.00141
1.0961	0.15406	1.5041	0.01192	2.2321	0.00134
1.1024	0.14357	1.5184	0.01124	2.2544	0.00127
1.1089	0.13386	1.5329	0.01060	2.2769	0.00121
1.1156	0.12487	1.5476	0.01000	2.2996	0.00115
1.1225	0.11653	1.5625	0.00943	2.3225	0.00109
1.1296	0.10879	1.5776	0.00890	2.3456	0.00104
1.1369	0.10161	1.5929	0.00841	2.3689	0.00099
1.1444	0.09493	1.6084	0.00794	2.3924	0.00094
1.1521	0.08873	1.6241	0.00750	2.4161	0.00090
1.1600	0.08297	1.6400	0.00708	2.4400	0.00085

z	$Q_{7/2}(z)$	z	$Q_{7/2}(z)$	z	$Q_{7/2}(z)$
2.4641	0.00081	3.5921	0.00013	5.0401*	0.00002
2.4884	0.00077	3.6244	0.00012	5.4521	0.00001
2.5129	0.00074	3.6569	0.00012		
2.5376	0.00070	3.6896	0.00011		
2.5625	0.00067	3.7225	0.00011		
2.5876	0.00064	3.7556	0.00010		
2.6129	0.00061	3.7889	0.00100		
2.6384	0.00058	3.8224	0.00100		
2.6641	0.00055	3.8561	0.00009		
2.6900	0.00053	3.8900	0.00009		
2.7161	0.00050	3.9241	0.00008		
2.7424	0.00048	3.9584	0.00008		
2.7689	0.00046	3.9929	0.00008		
2.7956	0.00044	4.0276	0.00007		
2.8225	0.00042	4.0625	0.00007		
2.8496	0.00040	4.0976	0.00007		
2.8769	0.00038	4.1329	0.00007		
2.9044	0.00036	4.1684	0.00006		
2.9321	0.00034	4.2041	0.00006		
2.9600	0.00033	4.2400	0.00006		
2.9881	0.00031	4.2761	0.00006		
3.0164	0.00030	4.3124	0.00005		
3.0449	0.00029	4.3489	0.00005		
3.0736	0.00027	4.3856	0.00005		
3.1025	0.00026	4.4225	0.00005		
3.1316	0.00025	4.4596	0.00005		
3.1609	0.00024	4.4969	0.00004		
3.1904	0.00023	4.5344	0.00004		
3.2201	0.00022	4.5721	0.00004		
3.2500	0.00021	4.6100	0.00004		
3.2801	0.00020	4.6481	0.00004		
3.3104	0.00019	4.6864	0.00004		
3.3409	0.00018	4.7249	0.00003		
3.3716	0.00018	4.7636	0.00003		
3.4025	0.00017	4.8025	0.00003		
3.4336	0.00016	4.8416	0.00003		
3.4649	0.00015	4.8809	0.00003		
3.4964	0.00015	4.9204	0.00003		
3.5281	0.00014	4.9601	0.00003		
3.5600	0.00013	5.0000	0.00003		

TABLE 4.8

$Q_{9/2}(z)$

$1.0001 \leq z \leq 3.2500$

$z = 1+y^2 \quad \Delta y = 0.01$

z	$Q_{9/2}(z)$	z	$Q_{9/2}(z)$	z	$Q_{9/2}(z)$
1.0001	2.76812	1.1681	0.03956	1.6561	0.00203
1.0004	2.08561	1.1764	0.03651	1.6724	0.00189
1.0009	1.69482	1.1849	0.03371	1.6889	0.00177
1.0016	1.42488	1.1936	0.03114	1.7056	0.00165
1.0025	1.22187	1.2025	0.02877	1.7225	0.00154
1.0036	1.06159	1.2116	0.02659	1.7396	0.00144
1.0049	0.93100	1.2209	0.02458	1.7569	0.00135
1.0064	0.82226	1.2304	0.02274	1.7744	0.00126
1.0081	0.73025	1.2401	0.02103	1.7921	0.00118
1.0100	0.65143	1.2500	0.01947	1.8100	0.00110
1.0121	0.58325	1.2601	0.01802	1.8281	0.00103
1.0144	0.52383	1.2704	0.01669	1.8464	0.00096
1.0169	0.47170	1.2809	0.01546	1.8649	0.00090
1.0196	0.42573	1.2916	0.01432	1.8836	0.00084
1.0225	0.38500	1.3025	0.01327	1.9025	0.00079
1.0256	0.34879	1.3136	0.01231	1.9216	0.00074
1.0289	0.31648	1.3249	0.01141	1.9409	0.00069
1.0324	0.28757	1.3364	0.01059	1.9604	0.00065
1.0361	0.26163	1.3481	0.00982	1.9801	0.00061
1.0400	0.23830	1.3600	0.00912	2.0000	0.00057
1.0441	0.21729	1.3721	0.00847	2.0201	0.00054
1.0484	0.19832	1.3844	0.00786	2.0404	0.00050
1.0529	0.18118	1.3969	0.00730	2.0609	0.00047
1.0576	0.16565	1.4096	0.00679	2.0816	0.00044
1.0625	0.15157	1.4225	0.00631	2.1025	0.00042
1.0676	0.13879	1.4356	0.00586	2.1236	0.00039
1.0729	0.12718	1.4489	0.00545	2.1449	0.00037
1.0784	0.11661	1.4624	0.00507	2.1664	0.00034
1.0841	0.10699	1.4761	0.00472	2.1881	0.00032
1.0900	0.09821	1.4900	0.00439	2.2100	0.00030
1.0961	0.09021	1.5041	0.00409	2.2321	0.00029
1.1024	0.08290	1.5184	0.00381	2.2544	0.00027
1.1089	0.07622	1.5329	0.00355	2.2769	0.00025
1.1156	0.07012	1.5476	0.00330	2.2996	0.00024
1.1225	0.06453	1.5625	0.00308	2.3225	0.00022
1.1296	0.05942	1.5776	0.00287	2.3456	0.00021
1.1369	0.05473	1.5929	0.00268	2.3689	0.00020
1.1444	0.05044	1.6084	0.00250	2.3924	0.00019
1.1521	0.04650	1.6241	0.00233	2.4161	0.00018
1.1600	0.04288	1.6400	0.00218	2.4400	0.00017

z	$Q_{9/2}(z)$		
2.4641	0.00016		
2.4884	0.00015		
2.5129	0.00014		
2.5376	0.00013		
2.5625	0.00012		
2.5876	0.00012		
2.6129	0.00011		
2.6384	0.00010		
2.6641	0.00010		
2.6900	0.00009		
2.7161	0.00009		
2.7424	0.00008		
2.7689	0.00008		
2.7956	0.00007		
2.8225	0.00007		
2.8496	0.00007		
2.8769	0.00006		
2.9044	0.00006		
2.9321	0.00006		
2.9600	0.00005		
2.9881	0.00005		
3.0164	0.00005		
3.0449	0.00004		
3.0736	0.00004		
3.1025	0.00004		
3.1316	0.00004		
3.1609	0.00004		
3.1904	0.00003		
3.2201	0.00003		
3.2500	0.00003		

TABLE 4.9

$Q_{11/2}(z)$

$1.0001 \leq z \leq 2.724$

$z = 1+y^2 \quad \Delta y = 0.01$

z	$Q_{11/2}(z)$	z	$Q_{11/2}(z)$	z	$Q_{11/2}(z)$
1.0001	2.58805	1.1681	0.02052	1.6561	0.00062
1.0004	1.90926	1.1764	0.01868	1.6724	0.00058
1.0009	1.52336	1.1849	0.01701	1.6889	0.00053
1.0016	1.25904	1.1936	0.01550	1.7056	0.00049
1.0025	1.06213	1.2025	0.01413	1.7225	0.00045
1.0036	0.90825	1.2116	0.01288	1.7396	0.00042
1.0049	0.78421	1.2209	0.01175	1.7569	0.00039
1.0064	0.68207	1.2304	0.01072	1.7744	0.00036
1.0081	0.59663	1.1240	0.00979	1.7921	0.00033
1.0100	0.52430	1.2500	0.00894	1.8100	0.00030
1.0121	0.46249	1.2601	0.00816	1.8281	0.00028
1.0144	0.40927	1.2704	0.00746	1.8464	0.00026
1.0169	0.36316	1.2809	0.00682	1.8649	0.00024
1.0196	0.32300	1.2916	0.00624	1.8836	0.00022
1.0225	0.28788	1.3025	0.00570	1.9025	0.00021
1.0256	0.25704	1.3136	0.00522	1.9216	0.00019
1.0289	0.22988	1.3249	0.00478	1.9409	0.00018
1.0324	0.20589	1.3364	0.00437	1.9604	0.00016
1.0361	0.18464	1.3481	0.00400	1.9801	0.00015
1.0400	0.16579	1.3600	0.00367	2.0000	0.00014
1.0441	0.14902	1.3721	0.00336	2.0201	0.00013
1.0484	0.13409	1.3844	0.00308	2.0404	0.00012
1.0529	0.12077	1.3969	0.00283	2.0609	0.00011
1.0576	0.10886	1.4096	0.00259	2.0816	0.00010
1.0625	0.09821	1.4225	0.00238	2.1025	0.00010
1.0676	0.08867	1.4356	0.00218	2.1236	0.00009
1.0729	0.08011	1.4489	0.00200	2.1449	0.00008
1.0784	0.07243	1.4624	0.00184	2.1664	0.00008
1.0841	0.06552	1.4761	0.00169	2.1881	0.00007
1.0900	0.05931	1.4900	0.00155	2.2100	0.00007
1.0961	0.05372	1.5041	0.00143	2.2321	0.00006
1.1024	0.04869	1.5184	0.00131	2.2544	0.00006
1.1089	0.04415	1.5329	0.00121	2.2769	0.00005
1.1156	0.04005	1.5476	0.00111	2.2996	0.00005
1.1225	0.03635	1.5625	0.00102	2.3225	0.00005
1.1296	0.03301	1.5776	0.00094	2.3456	0.00004
1.1369	0.02999	1.5929	0.00087	2.3689	0.00004
1.1444	0.02726	1.6084	0.00080	2.3924	0.00004
1.1521	0.02479	1.6241	0.00074	2.4161	0.00003
1.1600	0.02255	1.6400	0.00068	2.4400	0.00003

z	$Q_{11/2}(z)$		
2.4641	0.00003		
2.4884	0.00003		
2.5129	0.00002		
2.5376	0 00002		
2.7424	0.00001		

TABLE 4.10

$$Q_{13/2}(z)$$

$$1.0001 \leq z \leq 2.2100$$

$$z = 1+y^2 \quad \Delta y = 0.01$$

z	$Q_{13/2}(z)$	z	$Q_{13/2}(z)$	z	$Q_{13/2}(z)$
1.0001	2.43616	1.1681	0.01077	1.6561	0.00020
1.0004	1.76146	1.1764	0.00968	1.6724	0.00018
1.0009	1.38081	1.1849	0.00870	1.6889	0.00016
1.0016	1.12244	1.1936	0.00781	1.7056	0.00015
1.0025	0.93188	1.2025	0.00703	1.7225	0.00013
1.0036	0.78453	1.2116	0.00632	1.7396	0.00012
1.0049	0.66710	1.2209	0.00569	1.7569	0.00011
1.0064	0.57150	1.2304	0.00512	1.7744	0.00010
1.0081	0.49249	1.2401	0.00461	1.7921	0.00009
1.0100	0.42641	1.2500	0.00416	1.8100	0.00009
1.0121	0.37063	1.2601	0.00375	1.8281	0.00008
1.0144	0.32321	1.2704	0.00338	1.8464	0.00007
1.0169	0.28264	1.2809	0.00305	1.8649	0.00007
1.0196	0.24777	1.2916	0.00275	1.8836	0.00006
1.0225	0.21765	1.3025	0.00248	1.9025	0.00005
1.0256	0.19155	1.3136	0.00224	1.9216	0.00005
1.0289	0.16887	1.3249	0.00202	1.9409	0.00005
1.0324	0.14909	1.3364	0.00183	1.9604	0.00004
1.0361	0.13181	1.3481	0.00165	1.9801	0.00004
1.0400	0.11667	1.3600	0.00149	2.0000	0.00003
1.0441	0.10339	1.3721	0.00135	2.0201	0.00003
1.0484	0.09172	1.3844	0.00122	2.0404	0.00003
1.0529	0.08144	1.3969	0.00111	2.0609	0.00003
1.0576	0.07238	1.4096	0.00100	2.0816*	0.00002
1.0625	0.06439	1.4225	0.00091	2.2100	0.00001
1.0676	0.05732	1.4356	0.00082		
1.0729	0.05106	1.4489	0.00075		
1.0784	0.04552	1.4624	0.00068		
1.0841	0.04061	1.4761	0.00061		
1.0900	0.03625	1.4900	0.00056		
1.0961	0.03238	1.5041	0.00050		
1.1024	0.02894	1.5184	0.00046		
1.1089	0.02588	1.5329	0.00042		
1.1156	0.02315	1.5476	0.00038		
1.1225	0.02073	1.5625	0.00034		
1.1296	0.01856	1.5776	0.00031		
1.1369	0.01663	1.5929	0.00028		
1.1444	0.01491	1.6084	0.00026		
1.1521	0.01338	1.6241	0.00024		
1.1600	0.01200	1.6400	0.00021		

TABLE 4.11

$Q_{15/2}(z)$

$1.0001 \leq z \leq 1.8100$

$z = 1+y^2 \quad \Delta y = 0.01$

z	$Q_{15/2}(z)$	z	$Q_{15/2}(z)$	z	$Q_{15/2}(z)$
1.0001	2.30497	1.1681	0.00571	1.6561	0.00006
1.0004	1.63468	1.1764	0.00506	1.6724	0.00006
1.0009	1.25959	1.1849	0.00448	1.6889	0.00005
1.0016	1.00740	1.1936	0.00398	1.7056	0.00004
1.0025	0.82334	1.2025	0.00353	1.7225	0.00004
1.0036	0.68259	1.2116	0.00313	1.7396	0.00004
1.0049	0.57170	1.2209	0.00278	1.7569	0.00003
1.0064	0.48251	1.2304	0.00247	1.7744	0.00003
1.0081	0.40968	1.2401	0.00219	1.7921	0.00003
1.0100	0.34953	1.2500	0.00195	1.8100	0.00002
1.0121	0.29940	1.2601	0.00173		
1.0144	0.25732	1.2704	0.00154		
1.0169	0.22178	1.2809	0.00137		
1.0196	0.19163	1.2916	0.00122		
1.0225	0.16593	1.3025	0.00109		
1.0256	0.14395	1.3136	0.00097		
1.0289	0.12510	1.3249	0.00087		
1.0324	0.10888	1.3364	0.00077		
1.0361	0.09490	1.3481	0.00069		
1.0400	0.08282	1.3600	0.00061		
1.0441	0.07236	1.3721	0.00055		
1.0484	0.06329	1.3844	0.00049		
1.0529	0.05541	1.3969	0.00044		
1.0576	0.04856	1.4096	0.00039		
1.0625	0.04259	1.4225	0.00035		
1.0676	0.03738	1.4356	0.00031		
1.0729	0.03284	1.4489	0.00028		
1.0784	0.02887	1.4624	0.00025		
1.0841	0.02540	1.4761	0.00022		
1.0900	0.02236	1.4900	0.00020		
1.0961	0.01969	1.5041	0.00018		
1.1024	0.01736	1.5184	0.00016		
1.1089	0.01531	1.5329	0.00014		
1.1156	0.01351	1.5476	0.00013		
1.1225	0.01193	1.5625	0.00012		
1.1296	0.01054	1.5776	0.00010		
1.1369	0.00931	1.5929	0.00009		
1.1444	0.00823	1.6084	0.00008		
1.1521	0.00728	1.6241	0.00008		
1.1600	0.00645	1.6400	0.00007		

TABLE 4.12

$$Q_{17/2}(z)$$

$$1.0001 \leq z \leq 1.7396$$

$$z = 1+y^2 \quad \Delta y = 0.01$$

z	$Q_{17/2}(z)$	z	$Q_{17/2}(z)$	z	$Q_{17/2}(z)$
1.0001	2.18966	1.1681	0.00305	1.6561	0.00002
1.0004	1.52405	1.1764	0.00266	1.6724	0.00002
1.0009	1.15476	1.1849	0.00233	1.6889	0.00002
1.0016	0.90894	1.1936	0.00204	1.7056	0.00001
1.0025	0.73144	1.2025	0.00178	1.7225	0.00001
1.0036	0.59726	1.2116	0.00156	1.7396	0.00001
1.0049	0.49279	1.2209	0.00137		
1.0064	0.40979	1.2304	0.00120		
1.0081	0.34286	1.2401	0.00105		
1.0100	0.28828	1.2500	0.00092		
1.0121	0.24336	1.2601	0.00081		
1.0144	0.20615	1.2704	0.00071		
1.0169	0.17514	1.2809	0.00062		
1.0196	0.14916	1.2916	0.00055		
1.0225	0.12732	1.3025	0.00048		
1.0256	0.10889	1.3136	0.00042		
1.0289	0.09329	1.3249	0.00037		
1.0324	0.08005	1.3364	0.00033		
1.0361	0.06378	1.3481	0.00029		
1.0400	0.05918	1.3600	0.00025		
1.0441	0.05098	1.3721	0.00022		
1.0484	0.04396	1.3844	0.00020		
1.0529	0.03795	1.3969	0.00017		
1.0576	0.03279	1.4096	0.00015		
1.0625	0.02836	1.4225	0.00014		
1.0676	0.02455	1.4356	0.00012		
1.0729	0.02127	1.4489	0.00011		
1.0784	0.01844	1.4624	0.00009		
1.0841	0.01600	1.4761	0.00008		
1.0900	0.01389	1.4900	0.00007		
1.0961	0.01206	1.5041	0.00006		
1.1024	0.01048	1.5184	0.00006		
1.1089	0.00912	1.5329	0.00005		
1.1156	0.00794	1.5476	0.00004		
1.1225	0.00691	1.5625	0.00004		
1.1296	0.00602	1.5776	0.00004		
1.1369	0.00525	1.5929	0.00003		
1.1444	0.00458	1.6084	0.00003		
1.1521	0.00400	1.6241	0.00002		
1.1600	0.00349	1.6400	0.00002		

TABLE 4.13

$Q_{19/2}(z)$

$1.0001 \leq z \leq 1.5929$

$z = 1+y^2 \quad \Delta y = 0.01$

z	$Q_{19/2}(z)$	z	$Q_{19/2}(z)$
1.0001	2.08689	1.1681	0.00164
1.0004	1.42621	1.1764	0.00141
1.0009	1.06294	1.1849	0.00122
1.0016	0.82359	1.1936	0.00105
1.0025	0.65269	1.2025	0.00091
1.0036	0.52499	1.2116	0.00078
1.0049	0.42677	1.2209	0.00068
1.0064	0.34970	1.2304	0.00059
1.0081	0.28834	1.2401	0.00051
1.0100	0.23893	1.2500	0.00044
1.0121	0.19881	1.2601	0.00038
1.0144	0.16600	1.2704	0.00033
1.0169	0.13901	1.2809	0.00028
1.0196	0.11671	1.2916	0.00025
1.0225	0.09820	1.3025	0.00021
1.0256	0.08280	1.3136	0.00019
1.0289	0.06993	1.3249	0.00016
1.0324	0.05916	1.3364	0.00014
1.0361	0.05012	1.3481	0.00012
1.0400	0.04252	1.3600	0.00011
1.0441	0.03611	1.3721	0.00009
1.0484	0.03071	1.3844	0.00008
1.0529	0.02614	1.3969	0.00007
1.0576	0.02227	1.4096	0.00006
1.0625	0.01899	1.4225	0.00005
1.0676	0.01621	1.4356	0.00005
1.0729	0.01385	1.4489	0.00004
1.0784	0.01184	1.4624	0.00004
1.0841	0.01013	1.4761	0.00003
1.0900	0.08674	1.4900	0.00003
1.0961	0.00743	1.5041	0.00003
1.1024	0.00637	1.5184	0.00002
1.1089	0.00546	1.5329	0.00002
1.1156	0.00469	1.5476	0.00002
1.1225	0.00403	1.5625	0.00001
1.1296	0.00346	1.5776	0.00001
1.1369	0.00298	1.5929	0.00001
1.1444	0.00256		
1.1521	0.00220		
1.1600	0.00190		

TABLE 4.14

$$Q_{21/2}(z)$$

$$1.0001 \leq z \leq 1.5041$$

$$z = 1+y^2 \quad \Delta y = 0.01$$

z	$Q_{21/2}(z)$	z	$Q_{21/2}(z)$
1.0001	1.99432	1.1681	0.00088
1.0004	1.33878	1.1764	0.00075
1.0009	0.98168	1.1849	0.00064
1.0016	0.74888	1.1936	0.00054
1.0025	0.58454	1.2025	0.00046
1.0036	0.46320	1.2116	0.00039
1.0049	0.37102	1.2209	0.00034
1.0064	0.29960	1.2304	0.00029
1.0081	0.24345	1.2401	0.00024
1.0100	0.19884	1.2500	0.00021
1.0121	0.16308	1.2601	0.00018
1.0144	0.13422	1.2704	0.00015
1.0169	0.11080	1.2809	0.00013
1.0196	0.09170	1.2916	0.00011
1.0225	0.07607	1.3025	0.00009
1.0256	0.06323	1.3136	0.00008
1.0289	0.05265	1.3249	0.00007
1.0324	0.04391	1.3364	0.00006
1.0361	0.03668	1.3481	0.00005
1.0400	0.03068	1.3600	0.00004
1.0441	0.02569	1.3721	0.00004
1.0484	0.02154	1.3844	0.00003
1.0529	0.01808	1.3969	0.00003
1.0576	0.01519	1.4096	0.00002
1.0625	0.01278	1.4225	0.00002
1.0676	0.01076	1.4356	0.00002
1.0729	0.00906	1.4489	0.00002
1.0784	0.00764	1.4624	0.00001
1.0841	0.00645	1.4761	0.00001
1.0900	0.00545	1.4900	0.00001
1.0961	0.00459	1.5041	0.00001
1.1024	0.00388		
1.1089	0.00329		
1.1156	0.00278		
1.1225	0.00236		
1.1296	0.00200		
1.1369	0.00169		
1.1444	0.00144		
1.1521	0.00122		
1.1600	0.00104		

TABLE 4.15

$Q_{n-\frac{1}{2}}(z)$

$z = 1+y \qquad \Delta y = 0.1$

$1 \leq n \leq 11$

$1.1 \leq z \leq 5.00$*

*The upper limit of the tabulated arguments is governed by the condition $Q_{n-\frac{1}{2}}(z) \geq 0.00001$.

z	n = 1	n = 2	n = 3	n = 4
1.1	0.97877	0.48178	0.26070	0.14743
1.2	0.69956	0.28563	0.12870	0.06069
1.3	0.55365	0.19693	0.07745	0.03189
1.4	0.45980	0.14608	0.05138	0.01892
1.5	0.39318	0.11338	0.03622	0.01212
1.6	0.34302	0.09079	0.02664	0.00819
1.7	0.30373	0.07443	0.02023	0.00576
1.8	0.27204	0.06214	0.01575	0.00418
1.9	0.24592	0.05265	0.01251	0.00311
2.0	0.22401	0.04516	0.01010	0.00237
2.1	0.20537	0.03914	0.00828	0.00183
2.2	0.18932	0.03422	0.00687	0.00144
2.3	0.17536	0.03015	0.00576	0.00115
2.4	0.16312	0.02675	0.00488	0.00093
2.5	0.15229	0.02388	0.00416	0.00076
2.6	0.14266	0.02143	0.00358	0.00062
2.7	0.13404	0.01933	0.00310	0.00052
2.8	0.12628	0.01751	0.00270	0.00043
2.9	0.11926	0.01593	0.00237	0.00036
3.0	0.11289	0.01454	0.00208	0.00031
3.1	0.10708	0.01332	0.00184	0.00026
3.2	0.10177	0.01224	0.00164	0.00023
3.3	0.09689	0.01129	0.00146	0.00019
3.4	0.09239	0.01043	0.00131	0.00017
3.5	0.08825	0.00966	0.00118	0.00015
3.6	0.08441	0.00897	0.00106	0.00013
3.7	0.08084	0.00835	0.00096	0.00011
3.8	0.07752	0.00779	0.00087	0.00010
3.9	0.07443	0.00728	0.00079	0.00009
4.0	0.07154	0.00692	0.00073	0.00008
4.1	0.06884	0.00640	0.00066	0.00007
4.2	0.06630	0.00601	0.00061	0.00006
4.3	0.06392	0.00565	0.00056	0.00005
4.4	0.06168	0.00533	0.00051	0.00005
4.5	0.05957	0.00503	0.00047	0.00004
4.6	0.05758	0.00475	0.00044	0.00004
4.7	0.05570	0.00450	0.00041	0.00003
4.8	0.05392	0.00426	0.00038	0.00003
4.9	0.05223	0.00404	0.00035	0.00003
5.0	0.05063	0.00384	0.00032	0.00002

a	n = 5	n = 6	n = 7	n = 8
1.1	0.08558	0.05052	0.03019	0.01821
1.2	0.02941	0.01450	0.00724	0.00365
1.3	0.01351	0.00582	0.00254	0.00112
1.4	0.00717	0.00277	0.00108	0.00042
1.5	0.00417	0.00146	0.00052	0.00019
1.6	0.00259	0.00083	0.00027	0.00009
1.7	0.00169	0.00050	0.00015	0.00005
1.8	0.00144	0.00032	0.00009	0.00003
1.9	0.00080	0.00021	0.00006	0.00001
2.0	0.00057	0.00014	0.00003	0.00001
2.1	0.00042	0.00010	0.00002	0.00001
2.2	0.00031	0.00007	0.00002	
2.3	0.00024	0.00005	0.00001	
2.4	0.00018	0.00004	0.00001	
2.5	0.00014	0.00003	0.00001	
2.6	0.00011	0.00002		
2.7	0.00009	0.00002		
2.8	0.00007	0.00001		
2.9	0.00006	0.00001		
3.0	0.00005	0.00001		
3.1	0.00004			
3.2	0.00003			
3.3	0.00003			
3.4	0.00002			
3.5	0.00002			
3.6	0.00002			
3.7	0.00001			
3.8	0.00001			
3.9	0.00001			
4.0	0.00001			
4.1	0.00001			
4.2	0.00001			
4.3	0.00001			
4.4	0.00001			

z	n = 9	n = 10	n = 11
1.1	0.01105	0.00645	0.00414
1.2	0.00185	0.00094	0.00048
1.3	0.00050	0.00022	0.00010
1.4	0.00017	0.00007	0.00002
1.5	0.00007	0.00002	0.00001
1.6	0.00003	0.00001	
1.7	0.00001		
1.8	0.00001		

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